2022 UQ/QAMT Problem Solving Competition - Year 9 & 10 Paper

Two hours allowed. Rulers and non-CAS calculators may be used.
All questions have equal value with marks for working as well as correct answers.

Question 1

A group of 42 tourists visits a city which has exactly three monuments. At each monument, each tourist decides whether or not to take a photo of the monument. After the visit to the city, it turned out that any two tourists together have photos of all three monuments. What is the smallest number of photos that could have been taken by all the tourists in total?

Question 2

Suppose we have a triangle ABC such that the lines connecting B and C to the midpoints of the opposite sides meet at right angles. If the length of AB is 10 and the length of AC is 15, what is the length of BC?

Question 3

Suppose we have a sequence of numbers, t_n, such that $t_{2n} = t_n$ and $t_{2n+1} = 1 - t_n$. If $t_0 = 1$, is this sequence periodic?

Question 4

A right-angled triangle has sides of integer length. Its area (in square metres) is twice its perimeter (in metres). What are the lengths of the sides?

Question 5

Suppose we have a convex quadrilateral $ABCD$. M and N are points on AB and CD, respectively, such that $\frac{|AM|}{|AB|} = \frac{|CN|}{|CD|}$. Point P is the intersection of MD and AN, while point Q is the intersection of MC and BN. If the triangle BCQ has area 2 units and the triangle APD has area 3 units, what is the area of the quadrilateral $MPNQ$?