

2016 UQ/QAMT Problem Solving Competition - Year 9 & 10 Paper

Two hours allowed. All questions have equal value. Non-CAS calculators may be used.

Question 1

In 2016 the first day of lectures at the University of Queensland was Monday, February 29th. When are the next three years in which there is a February 29th falling on a Monday?

Question 2

Suppose $a \leq b \leq c$ are positive integers. We have abc identical small cubes, and used them to build an $a \times b \times c$ brick. Suppose the number of small cubes with at least one external face is equal to the number of totally enclosed cubes. If $a = 8$, what is the largest possible value of abc ?

Question 3

You have eight identical looking batteries. You know that four of them are good and four are bad, but you do not know which are which. You need two good batteries for your personal destructor beam gun. You can test them only by placing two batteries into the gun and seeing if the device works. It will function only with two good batteries.

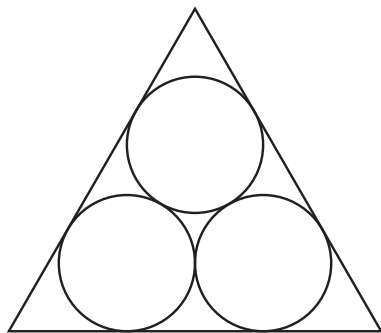
How many tests are necessary before you can be sure to identify two good batteries?

Question 4

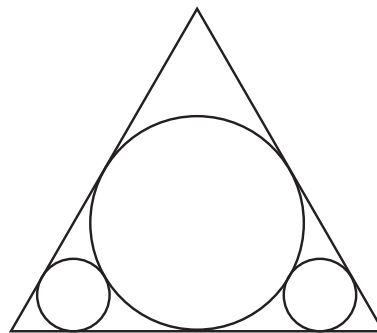
What are the first two integers greater than 2000 with exactly 36 divisors each?

Question 5

Suppose we have two equilateral triangles, each with side length 1, as shown below. For triangle (a), three circles of equal radius are drawn inside the triangle. Each circle is tangent to the other two, and to two of the sides of the triangle. Triangle (b) also has three circles, the largest circle that can be drawn inside the triangle and two smaller circles with equal radii that are tangent to the large circle and to two sides of the triangle. Is the area covered by the circles bigger in (a) or (b), or are they the same? Find values for the areas and compare.



(a)



(b)

