THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

2012 UQ/QAMT Problem Solving Competition - Year 9 & 10 Paper

All questions have equal value.

Question 1
If a is a real number, $\lfloor a \rfloor$ denotes the largest integer n with $n \leqslant a$, and $\{a\}$ denotes the number r with $0 \leqslant r < 1$ for which $a - r$ is an integer. For example $\lfloor \frac{9}{4} \rfloor = 2$ and $\{\frac{9}{4}\} = \frac{1}{4}$. Which values of x (if any) satisfy

$$2\lfloor x \rfloor = x + 2\{x\}\ ?$$

Question 2
Starting with a positive integer n, form the sum of decimal digits of n, then form the sum of digits of this new number and so on, until the process stabilizes. The result is called the *ultimate digital sum* of n. What is the ultimate digital sum of 2^{2012}?

Question 3
A car, van, truck and bike are all travelling in the same direction on the road. Each travels at a constant speed, but the speeds of the 4 vehicles may be different. At 10 am the car overtakes the van. At noon it overtakes the truck. At 2 pm it overtakes the bike. At 4 pm the truck overtakes the bike. At 6 pm the van overtakes the truck. When does the van overtake the bike?

Question 4
Two congruent non-overlapping equilateral triangles are placed wholly within a square of side length 1. What is the maximum area they could cover?

Question 5
Suppose x, y and z are numbers satisfying

$$\begin{aligned} x+y+z &= -2 \\ x^2+y^2+z^2 &= 122 \\ x^3+y^3+z^3 &= 142 \end{aligned}$$

What is the value of xyz?