2011 UQ/QAMT Problem Solving Competition - Year 9 & 10 Paper

All questions have equal value.

Question 1
Find all real numbers x such that
$$\frac{27^x + 8^x}{12^x + 18^x} = \frac{7}{6}$$

Question 2
There are four houses on a street. Each house is a different colour, blue, red, white or black. Each house is inhabited by a different animal and has a different type of tree growing outside it. The tortoise lives in house three while the dog lives directly next to the black house. The pine tree grows in front of house four, the gum tree grows in front of house two and the maple tree grows directly next to the dog's house. The second house is red and the first house is not blue. If the horse lives in house two, what colour is the butterfly's house?

Question 3
Consider a triangle ABC with total area 26 cm^2. Let X be the midpoint of BC and Y be the midpoint of AC. Suppose Z is chosen on the line AB such that the area of AZY is 9 cm^2. What is the area of XYZ?

Question 4
Let a, b and c be distinct non-zero real numbers such that
$$a + \frac{1}{b} = b + \frac{1}{c} = c + \frac{1}{a}.$$
What is the value of $|abc|$?

Question 5
Draw n equally spaced marks between 0 and 1 on each of the x and y axes. Connect the first mark on the x-axis (closest to 0) to the last mark on the y axis (closest to 1), the second mark on the x axis to the second last on the y axis, and so on.

As n increases this process creates a curved boundary, C, as seen in this example diagram. Give an equation for this curve as $n \to \infty$.