THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

2008 UQ/QAMT Problem Solving Competition - Year 9 & 10 Paper

All questions have equal value.

Question 1
What are all the values of n such that $2^n + 2^{11} + 2^8$ is a perfect square?

Question 2
ABC is a triangle with $|AB| = |AC|$ and $\angle BAC = 20°$. E is a point on AB with $\angle BCE = 50°$ and D is point on AC with $\angle CBD = 40°$. What is the angle $\angle CED$?

Question 3
A positive integer n has first (decimal) digit 9. If this digit is moved to the end, the resulting integer is exactly $n/7$. What is the smallest n satisfying this condition?

Question 4
A real-valued function f is defined on the integers, and satisfies:

(1) $f(0) = 1$
(2) $f(x + y + 1) + 3 = f(x + 1) + f(y + 1) + 2xy$

What is the $f(45) - f(4)$.

Question 5
Two people, A and B, agree to play the following game. A barrel is filled with a large number of balls. Half the balls are labelled A and half B. The players take it in turns to draw a ball from the barrel. If it is labelled with their name they win and the game ends. Otherwise they replace their ball and the other player draws. If A goes first, what is the probability that A wins?

Question 6
Which positive integers can be written as the sum of two or more *consecutive* positive integers? For example $6 = 1 + 2 + 3$ but 8 cannot be thus written.