

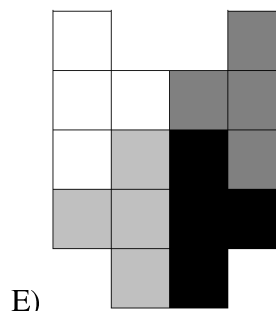
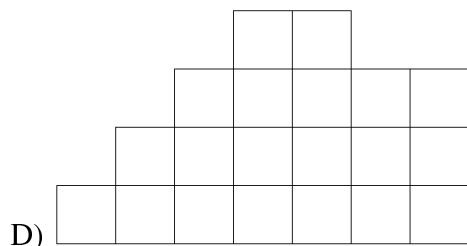
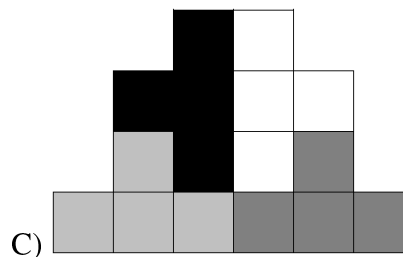
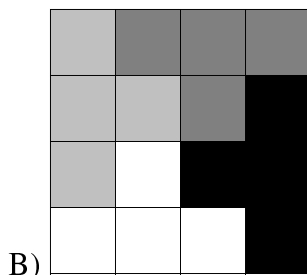
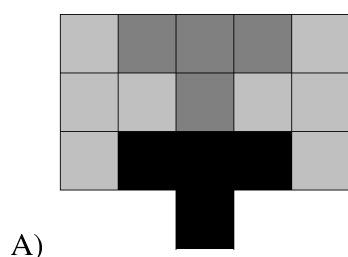
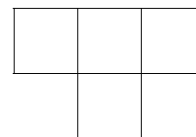


2002 QAMT Competition Year 9&10 Paper



Solutions

Q1. (1 point) Which shape cannot be filled, without any overlapping, using copies of the tile shown on the right?



Solution: The diagrams above show how you can fill in the shapes except for (D).

Q2. (1 point) We want to fill in the remaining squares in such a way that each of the numbers 1, 2, 3, 4, 5 and 6 appears in every row and every column. In how many ways can this be done?

1	2	3	4	5	6
2					5
3					4
4					3
5					2
6	5	4	3	2	1

- (A) 16, (B) 24, (C) 2^{16} , (D) 24^4 , (E) 16^2 .

Solution: Start with entries where there is no choice and work through the entries with several possibilities. So first fill in the entry I row 2, column 3. There is already a 2 and a 5 in the row and a 3 and a 4 in the column, so the only choices are 1 and 6. Choosing 1 means three

other entries follow. Choose 6, then the other three entries follow. So this means that the choices are:

1	2	3	4	5	6
2		1	6		5
3					4
4					3
5		6	1		2
6	5	4	3	2	1

Or

1	2	3	4	5	6
2		6	1		5
3					4
4					3
5		1	6		2
6	5	4	3	2	1

Filling in the four central entries, there are again two choices for each time. So for the first choice above we get either of

1	2	3	4	5	6
2		1	6		5
3		2	5		4
4		5	2		3
5		6	1		2
6	5	4	3	2	1

Or

1	2	3	4	5	6
2		6	1		5
3		5	2		4
4		2	5		3
5		1	6		2
6	5	4	3	2	1

The remaining entries follow the same pattern. Each new group gives two choices. So, there are $2^4 = 16$ different ways of completing the square, and the answer is (A).

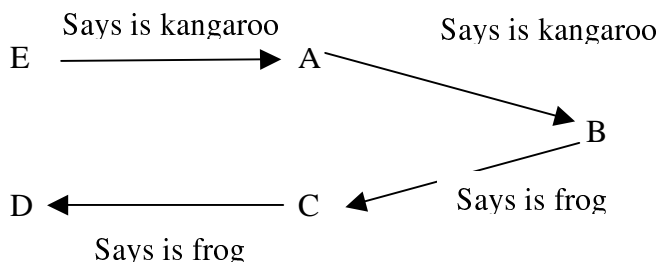
Q3. (2 points) Adam, Brett, Chad, Dan and Eve play a game in which each is a frog or kangaroo. Frogs always tell lies while kangaroos always tell the truth.

- Adam says that Brett is a kangaroo.
- Chad says that Dan is a frog.
- Eve says that Adam is not a frog.
- Brett says that Chad is not a kangaroo.
- Dan says that Eve and Adam are different kinds of animals.

How many frogs are there?

- (A) 1, (B) 2, (C) 3, (D) 4, (E) 5

Solution: We have the following situation:



Assume Eve is a kangaroo and hence that her statement is true. Then Adam is a kangaroo, Brett is a kangaroo, Chad is a frog and Dan is a kangaroo. But this is not possible since Eve and Adam are then both kangaroos, contradicting Dan’s statement. This proves that Eve is not a kangaroo, but a frog instead.

Being a frog, Eve’s statement is false. Thus Adam is a frog, Brett is a frog, Chad is a kangaroo and Dan is a frog. Thus there are 4 frogs.

Q4. (2 points) What is the sum:

$$\frac{1}{2} + \left(\frac{1}{3} + \frac{2}{3}\right) + \left(\frac{1}{4} + \frac{2}{4} + \frac{3}{4}\right) + \left(\frac{1}{5} + \frac{2}{5} + \frac{3}{5} + \frac{4}{5}\right) + \dots + \left(\frac{1}{100} + \frac{2}{100} + \frac{3}{100} + \dots + \frac{98}{100} + \frac{99}{100}\right)?$$

- (A) 105, (B) 245, (C) 2475, (D) 3215, (E) 2635

Solution:

$$\begin{aligned} &\frac{1}{2} + \left(\frac{1}{3} + \frac{2}{3}\right) + \left(\frac{1}{4} + \frac{2}{4} + \frac{3}{4}\right) + \left(\frac{1}{5} + \frac{2}{5} + \frac{3}{5} + \frac{4}{5}\right) + \dots + \left(\frac{1}{100} + \frac{2}{100} + \frac{3}{100} + \dots + \frac{98}{100} + \frac{99}{100}\right) \\ &= \frac{1}{2}(1) + \frac{1}{2}(2) + \frac{1}{2}(3) + \dots + \frac{1}{2}(99) \end{aligned}$$

The n^{th} bracketed term contains n terms whose average value is $\frac{1}{2}$.

$$\begin{aligned} &= \frac{1}{2}(1 + 2 + 3 + \dots + 99) \\ &= \frac{1}{2} \times \frac{99}{2}(1 + 99) \\ &= \frac{9900}{4} \\ &= 2475 \end{aligned}$$

Q5. (2 points) In the addition sum

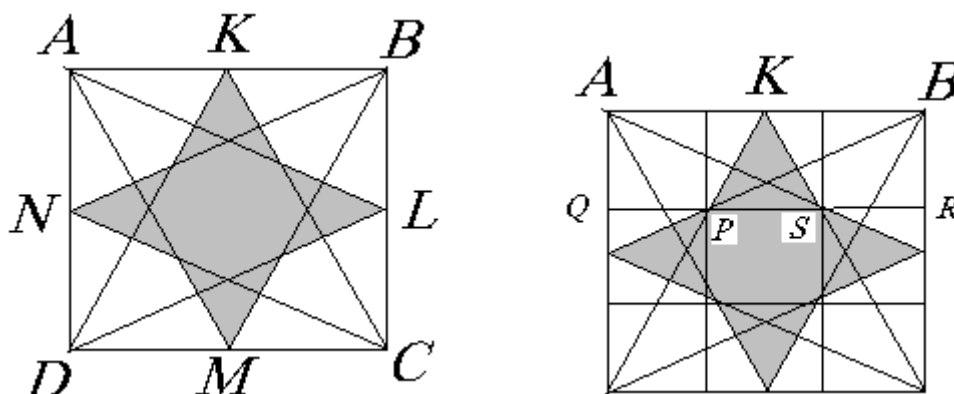
$$\begin{array}{r} T W O \\ + T W O \\ \hline F O U R \end{array}$$

The letters F, O, R, T, U, W stand for digits 1, 2, 3, .. 9 (but not zero) with different letters standing for different digits. Find the values of F, O, R, T, U and W that make the number $F O U R$ as small as possible.

Solution: Since the digit 0 is not allowed, in $FOUR$ we must have $F=1$ and hence $T \geq 5$. For $FOUR$ to be least, first try with $T=5$. But then we would have $O = 0$ or $O = 1$, neither of which were allowed. Next try with $T=6$ giving $O = 2$ or $O=3$. If $O = 2$, then $R=4$ and $W=3$, since $W \neq 1, 2, 4$) which would make $U = 6 = T$ and is not allowed. If $O=3$, then $R=6=T$ and is not allowed. So we cannot have $T=6$. Now try $T=7$, so $O=4$ (since $O=5$ makes $R=0$). Then $W \neq 1, 4$. With $W=2$, we have $U=4=O$, but with $W=3$ we get $U=6$ which is fine. Thus, the least solution is

$$\begin{array}{r} 734 \\ + 734 \\ \hline 1468 \end{array}$$

Q6. (2 points) This four-pointed star is formed by taking a square with side length 1 metre and joining the midpoints of each side to the corners as shown. What is the area of the star?



Solution: Divide the square ABCD into nine regions as shown. Of course, $PQ = 1/3 AB$, so all the small regions are squares. Since K is the midpoint of AB, triangle KPS has half the area of its small square. The star is made up of four triangles like KPS plus the small central square, so its area equals that of three small squares and hence is $1/3$ the area of square ABCD = 0.333 square metres.

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