2009 UQ/QAMT Problem Solving Competition - Year 8 Paper

All questions have equal value.

Question 1
What are the last two digits in the number 7^{2009}?

Question 2
P is a point inside an equilateral triangle. The distances from P to each of the sides are 5, 8 and 19. What is the height of the triangle?

Question 3
Find a 3 digit prime number that remains prime under all rearrangements of its digits. For example, 157 is prime but 715 is not, so 157 is no good.

Question 4
Write the date as dd/mm/yy. How many times in a century does dd $\times$ mm $=$ yy?

Question 5
An island is inhabited only by angels and demons. Angels always tell the truth, and demons always lie.

You meet three inhabitants: Anne, Bill and Chris. Bill claims, "I am a angel or Anne is a demon." Chris says, "Exactly one of Anne and myself is a angel." Anne tells you that Chris is a demon.

Which of Anne, Bill and Chris are angels?

Question 6
You have a bottle and a jug, both containing orange juice. The bottle is one third full, and the jug is one sixth full. Water is added to both containers until they are full, and they are then both poured into a bucket. If the mixture in the bucket is one fifth orange juice, and the capacity of the bottle is 1 litre, what is the capacity of the jug?