2008 UQ/QAMT Problem Solving Competition - Year 8 Paper

All questions have equal value.

Question 1
How can you arrange 9 dots in the plane so that there are 9 different straight lines connecting 3 of the dots?

Question 2
A positive integer n has first (decimal) digit 7. If this digit is moved to the end, the resulting integer is exactly $n/4$. What is the smallest n satisfying this condition?

Question 3
A (large) sheet of paper contains the following statements:
  1   Exactly 1 statement on this page is false
  2   Exactly 2 statements on this page are false
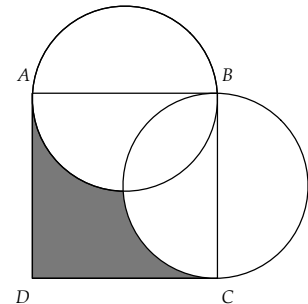 ⋮   ⋮
  100   Exactly 100 statements on this page are false
How many statements on this sheet of paper are false?

Question 4
ABCD is a square with side length 8 cm. Circles are drawn with diameters AB and BC, giving the diagram to the right.

What is the area inside the square not covered by either of the circles?

Question 5
A jail has 48 cells, labelled 1 to 48. All are initially locked. The jailer makes 48 rounds of turning keys. In each round he begins at cell 1 and proceeds in order to cell 48. In the first round he turns every key. In the second round he turns every second key. In the third round he turns every third key, and so on. In the last round he turns just the 48th key. After this process, how many cells are unlocked?

Question 6
What are all the values of n such that $2^n + 2^{11} + 2^8$ is a perfect square?