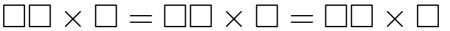




2005 QAMT Problem-Solving Competition - Year 8 Paper

Question 1 Using the numbers 1 to 9 each once, fill in the boxes to make the equation correct.



Solution

 $18 \cdot 9 = 27 \cdot 6 = 54 \cdot 3.$

Question 2 Arrange 6 matches of equal length to form 8 equilateral triangles. The matches must not be broken and must be laid end to end, although they are allowed to cross.

3 marks

2 marks

Solution

The Star of David is one solution.

Question 3 What are the largest and smallest number of Friday the 13ths that can occur in any year (Jan 1st to Dec 31st)?

Solution

February 13th is always $31 = 4 \cdot 7 + 3$ days after January 13th, so is offset by being 3 days later in the week. Extending this calculation we obtain the following offsets:

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Non-leap												
Leap												

The number of times each offset occurs is

	0	1	2	3	4	5	6	
Non-leap	2	1	1	3	1	1	2	
Leap	3	1	1	2	2	1	2	

Since 0, 1, ..., 6 all occur at least once in each case, no matter which day January 13th falls on there will always be at least one Friday the 13th. The maximum possible number is 3, which occurs in non-leap years if January 13th is a Tuesday, and in leap years if January 13th is a Friday.

Question 4 According to a survey, at least 70% of people like apples and at least 75% like bananas. What can you say about the percentage of people who like both?

4 marks

4 marks

Solution

Let a be the percentage who like apples but not bananas, b who like bananas but not apples, and c the percentage who like both. Then we want to maximize/minimize c given constraints:

$$a, b, c \ge 0$$
$$a + b + c \le 100$$
$$a + c \ge 70$$
$$b + c \ge 75$$

The second inequality gives $c \le 100 - (a + b) \le 100$ and c = 100, a = b = 0 is a valid solution. Adding the last two inequalities, $a + b + 2c \ge 145$, or $-a - b - 2c \le -145$. Adding the second inequality, $-c \le -45$ or $c \ge 45$. And c = 45, a = 25, b = 30 is a valid solution. So

$$45 \leq c \leq 100.$$

Problems of this type are called *linear programming* problems.

Question 5 Suppose you have four rectangles of size 1×2 , three of size 2×3 , two of size 3×3 , and one of size 5×5 . Can you arrange them to form a single large rectangle?

4 marks

Solution

No. The total area is $69 = 3 \cdot 23$, so the only possible dimensions for the large rectangle would be 3×23 or 1×69 , and the 5×5 square will not fit in either.

Question 6 Suppose x and y are positive integers satisfying 13x + 8y = 2005. What is the minimum possible value of x + y?

3 marks

Solution

2005 = 8(x + y) + 5x so we should make x as large as possible. But $13x \le 2005 - 8$ and x is an integer, so $x \le 153$. If x = 153, y = 2 so the smallest x + y can be is 155, with x = 153, y = 2.



