

2004 QAMT Problem-Solving Competition - Year 8 Solutions

Question 1 Which of the figures below can **not** be folded to make a cube?

2 marks

Solution The top right figure only.

Question 2 3 bushwalkers meet at a camping ground. Bushwalker A has 5 chocolate bars, B has 3 and C has none, but C has \$8. A and B agree to distribute their chocolate bars so that everyone has an equal share; C agrees to pay for the chocolate bars with his \$8. What is the fair amount that C should pay A?

2 marks

Solution \$7.

Everyone now has $\frac{8}{3}$ chocolate bars: A has contributed $\frac{7}{3}$ bars to C, and B has only contributed $\frac{1}{3}$ of a bar. So A has given away 7 times as much chocolate as B, so should receive \$7 to B's \$1.

Question 3 Find a four digit even number $abcd$ (where a, b, c, d are the four digits) such that 1.5 times $abcd$ equals the four digit number $dcba$. ($a = 0$ is not allowed.)

4 marks

Solution 4356.

Let n be the number. d is even and not 0, so $d \in \{2, 4, 6, 8\}$ while $a < 7$ (else $1.5n$ has five digits). The condition is $abcd + abcd + abcd = dcba + dcba$. The last digit of $3d$ is the same as the last digit of $2a$. Let $3d - 2a = 10z$. Then $10z < 3d < 30$ and $10z > -2a > -14$, so $z \in \{-1, 0, 1, 2\}$. Next write $3a + x = 2d + y$ where x and y represent possible carries: $x \in \{0, 1, 2\}$, $y \in \{0, 1\}$. Multiplying by 2, $4d + 2y = 6a + x = 9d - 30z$ so $5(d - 6z) = 2(y - x)$. Hence 5 divides $y - x$, so $y = x$ and $d = 6z$. Since d is a non-zero digit we have $d = 6$. Now $2dcba$ is between 12000 and 14000, so $a = 4$. Looking at the middle digits we need $2cb = 3bc + 1$, ie $20c + 2b = 30b + 3c + 1$, so $17c = 28b + 1$, so c is odd. Testing $c = 1, 3, 5, 7, 9$, the only possibility is $b = 4, c = 5$. So $n = 4356$. Check: $1.5 \cdot 4356 = 6534$.

Question 4 Show how to place 12 matches of unit length to form the perimeter of a single polygon to enclose exactly 4 square units of area. No matches may be placed inside the polygon. No match may be broken, or placed on top of another match and each match must be laid end to end with other matches.

4 marks

Solution There may be many solutions.

The 3-4-5 triangle has perimeter 12, but area 6. The desired polygon can be made by "indenting" this triangle by two squares: move three of the matches nearest the right angle nearest the right angle to cut an area of 2 squares out of the triangle.

Question 5 Professor Kato has forgotten her 8 digit phone number. But she remembers that the first digit is the number of 0's in the number, the second digit is the number of 1's, the third digit is the number of 2's and so on. What is her phone number?

4 marks

Solution 42100100

This is found by a combination of trial and error and reasoning.

Let the entry in the i th position be a_i , $i = 0, 1, \dots, 7$. It's clear that most of the entries will have to be 0. For example, if a_4 and a_5 are both 1 the list must contain a 4 and a 5, so four of something and five of something else, which already means at least 9 entries are needed. Indeed, the same argument shows a_4, \dots, a_7 contains at most one non-zero entry, which can only be a 1. (The non-obvious case is if $a_4 = 2$. Then we have exactly two 4's in our list and a 2, but the two fours mean the list has four entries of something and four of something else; these entries can only be 2's and 4's, so the entire list must consist of 4's and 2's but then $a_4 = 4$ not 2.)

On the other hand, the four entries a_4, \dots, a_7 cannot all be zero, since then we would have at least 4 zeros, so $a_0 \geq 4$ and hence one of a_4, \dots, a_7 would have to be at least 1 to indicate this. So a_4, \dots, a_7 contains a 1 and three zeros. Thus $a_0 \geq 3$, and $a_1 \geq 1$. The list has at least 3 non-zero entries, so $a_0 = 3, 4$ or 5 . And so on. Eventually we get the solution 42100100.

Question 6 An astronaut comes across four different species of Martians. One species always lies. One always tells the truth. One sometimes lies and sometimes tells the truth. The final species always lies and also refuses to answer any questions. The Martians always travel in groups of four, with one of each species present. Unfortunately the astronaut cannot tell the Martians apart! One day he comes across four Martians (one of each species) called Andrew, Betty, Charles and Dianne. They say the following things to him:

Andrew says: "Talk to Dianne. She always tells the truth." Betty says: "Don't talk to me. I never answer questions." Charles says: "That's true. Betty never answers questions."

Which one of the four always tells the truth?

4 marks

Solution D.

Call the four species the liars, the truth tellers, the middlemen and the refusers.

Despite her claim, B cannot be a refuser, for if she were, she would also lie about it. Thus B and C are both lying. So one of A or D must be the truth teller.

Suppose A is lying. Then D sometimes lies also, and no-one can be the truth teller. So A must be telling the truth—hence D is the truth teller. (A is the middleman and B, C are the liar and refuser but we don't know which is which.)