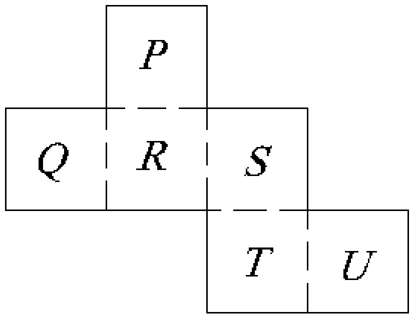
2002 QAMT Competition
Year 8 Paper

Solutions

Q1. (1 point) When the diagram shown is folded to make a cube, then the face marked *U* is opposite to the face marked

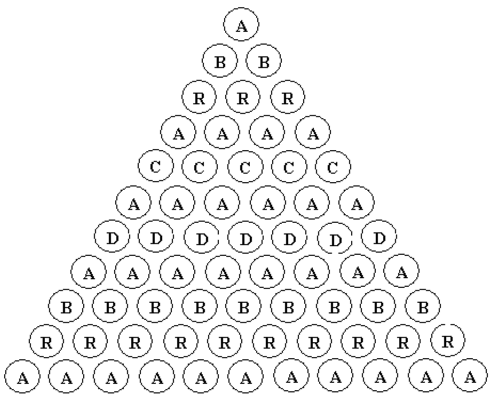
(A) *P* (B) *Q* (C) *R* (D) *S* (E) *T*



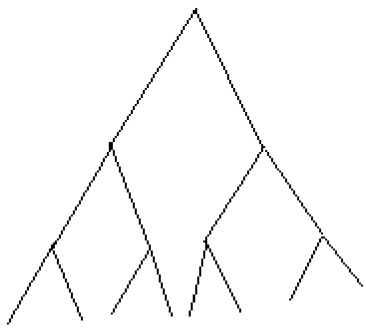
Solution: The answer is (C) *R*. One solution method is cutting out the figure and making the cube! Otherwise, you can just visualise it.

Q2. (1 point) In how many ways can the word ABRACADABRA be spelt out, using adjacent letters in the arrangement below?

(A) 100 (B) 144 (C) 512 (D) 1024 (E) More than 1024



Solution: The word abracadabra starts with ‘a’, so it could start at the top or bottom. But if the ‘a’s at the bottom are not adjacent to any ‘b’s, so the first letter must be at the top of the pyramid. In fact, abracadabra has a special structure: if you start at the ‘a’ at the top, you must always be moving down with each step. This means you can treat the problem like a tree:



There are 10 branches, and hence there are $2^{10} = 1024$ different ways of spelling out abracadabra and hence (D) is the answer.

Q3. (1 point) My Mum drives me from home to school each morning and then she returns home. She takes the same route in both directions and does not stop. On the way to school her average speed is 60 km/h and on the way home her average speed is 40 km/h. Which of the following is her average speed for the entire trip?

(A) 45 km/h (B) 48 km/h (C) 50 km/h (D) 52 km/h (E) 48.99 km/h

Solution: Speed is measured in km/h, so in other words, speed = distance/time. If the distance is d, speed is s and time is t, then $s = d/t$. For the forward trip, the distance is $d = 60\ t_1$. For the return trip, the distance is $d = 40\ t_2$. Now, the average speed is the total distance divided by the total time:

$\bar{s} = \dfrac{2d}{t_1 + t_2} = \dfrac{2d}{d/60 + d/40} = \dfrac{2}{1/60 + 1/40} = 48$ and the answer is (B). There are many different ways to find an 'average': the arithmetic mean, the harmonic mean or the geometric mean. You can investigate these further on your own.

Q4. (2 points) A group of year 8 students in Longreach organised a car wash. Some cars had a basic wash for \$5 each while the rest had a deluxe wash for \$7. A total of \$176 was raised. What was the minimum number of cars washed?

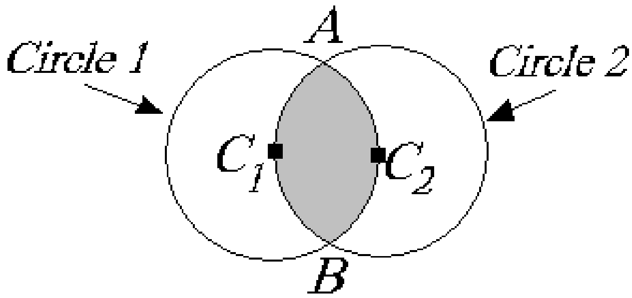
(A) 23 (B) 24 (C) 26 (D) 28 (E) 30

Solution: The number of customers with a \$5 basic wash is x and the number of customers with a deluxe wash is y. Then $5x + 7y = 176$. The minimum of $x+y$ will happen when x is least. Now,

$7y = 176 - 5x$ and so $y = 25 + (1\text{-}5x)/7$

and the smallest x which lets y be an integer is x=3, giving $y = 23$, and the minimum number of customers is 23+3 = 26.

Q5. (2 points) The diagram below shows two circles, Circle 1 and Circle 2. The centre *C1* of circle 1 lies on the circumference of Circle 2, and the centre *C2* of Circle 2 lies on the circumference of Circle 1. The two circles intersect at points *A* and *B*. The length of the line joining *C1* and *C2* is 2 cm, and the length of the line joining *A* and *B* is $2\sqrt{3}$ cm. Find the area common to both circles (shaded).



Solution: The area of the shaded region AC_1BC_2 must be two times the area of segment AC_1B and this corresponds to two times the area of sector C_2AB minus the area of triangle ABC_2, since triangle C_1AC_2 is equilateral.

The area of sector C_2AB is 1/3 the area of the circle = $1/3 \times \pi \times 2^2 = 4\pi/3$.

The area of triangle $ABC_2 = ½ (2\sqrt{3} \times 1) = \sqrt{3}$

Hence the shaded area is $2(4\pi/3 - \sqrt{3}) = 8\pi/3 - 2\sqrt{3}$.

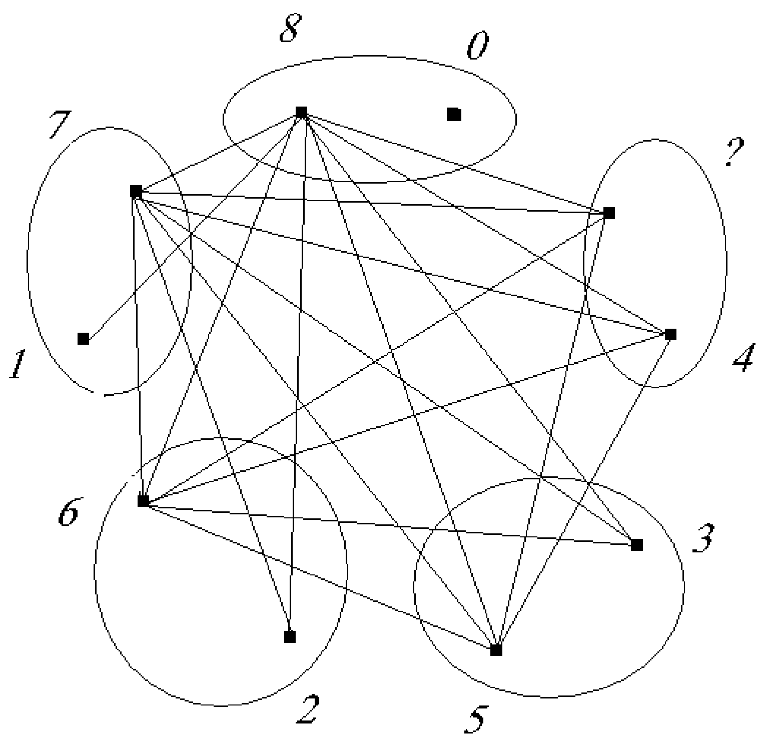
Q6. (3 points) A tennis tournament had five teams of two people:

- Fred & Alice
- Jayne & David
- Shen & Felicity
- Lynne & Brian
- Gina & Richardo

However, not all of the games were friendly and not all the players shook hands. In fact, no player shook his or her own hand, no player shook the hand of their partner, only some players shook their opponents' hands and no two players shook hands more than once. At the end of the tournament, Richardo asked each player how many hands he or she had shaken. Every player gave a different answer, except Gina had the same answer as Richardo. How many hands did Gina shake?

Solution: Consider a graph connecting vertices. The *vertices* represent each player, and the *links* on the graph represent those who shook hands with others. There must be 10 vertices (as there were 10 players) and they must have different *degrees*. (The degree is the incident number of links at a vertex.). There are 10 players and no player shook their own hand and no player shook their partners hand. So each player can shake at most 8 hands. Since there are 9 players excluding Ricardo and each player shakes a different number of hands, they must shake 0, 1, 2, 3, 4, 5, 6, 7 or 8 hands. In other words, the degrees must be 0, 1, 2, 3, 4, 5, 6, 7, 8 and one more vertex of unknown degree for Ricardo. The vertices must be matched as shown below – you can build the graph by starting with a vertex with degree 8, then 7, then 6, etc.

It turns out that the last vertex must have degree 4. This must be Ricardo's and Gina's degree, and the number of hands that they shook.


8
0
7
?
1
4
6
3
2
5