THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

2022 UQ/QAMT Problem Solving Competition - Year 11 & 12 Paper

Two hours allowed. Rulers and non-CAS calculators may be used.
All questions have equal value with marks for working as well as correct answers.

Question 1

What four digit numbers can be obtained from $p^4-p^3-p^2+2p-1$, where p is a prime number?

Question 2

For what positive integers n can an $n \times n$ table be filled with the numbers $1, 2, -3$ such that the sum of the entries in each row and in each column is zero?

Question 3

Consider the equation

$$f(f(f(f(f(f(f(f(f(f(x)))))))))) + \frac{1}{2} = 0,$$

where $f(x) = |x| - 1$ has been applied 10 times. How many values of x satisfy this equation?

Question 4

What integers m and n satisfy the equation $2^{2m+1} + 9 \cdot 2^m + 5 = n^2$?

Question 5

Suppose we have a convex quadrilateral $ABCD$. M and N are points on AB and CD, respectively, such that $\frac{|AM|}{|AB|} = \frac{|CN|}{|CD|}$. Point P is the intersection of MD and AN, while point Q is the intersection of MC and BN. If the triangle BCQ has area 2 units and the triangle APD has area 3 units, what is the area of the quadrilateral $MPNQ$?