THE UNIVERSITY
OF QUEENSLAND
A U S T R A L I A

# 2021 UQ/QAMT Problem Solving Competition - Year 11 & 12 Paper

Two hours allowed. Rulers and non-CAS calculators may be used.
All questions have equal value with marks for working as well as correct answers.

Question 1

Is it possible to colour all positive integers with red or blue so that any two numbers differing by 5 are different colours, and doubling any number gives a number of a different colour? If not, what is the smallest n where it is not possible to colour the first n positive integers in this way?

Question 2

Suppose the square $ABCD$ is inscribed in a circle. Four semicircles are then drawn with each of the square's sides as a diameter, as shown in the following diagram. If the perimeter of the square is 48 cm, what is the area of the shaded region?
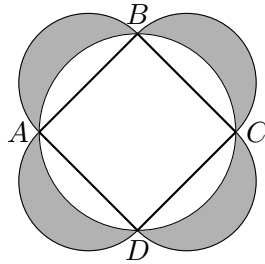
Question 3

Let N be the largest number with 2021 digits such that $7N$ also has 2021 digits. What are the last five digits of N?

Question 4

Anna writes the sum
$$1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{1000}$$
as a simplified fraction $\frac{a}{b}$ (so that a and b have no common divisor). How many zeros are at the end of b?

Question 5

A positive integer is called "charming" if the sum of its digits is divisible by 31. What is the maximum possible difference between two consecutive charming numbers?