THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

2016 UQ/QAMT Problem Solving Competition - Year 11 & 12 Paper

Two hours allowed. All questions have equal value. Non-CAS calculators may be used.

Question 1
Consider the (decimal) numbers 10, 101, 1010, 10101, ... in which the digits 1 and 0 alternate. What is the largest prime number in this sequence?

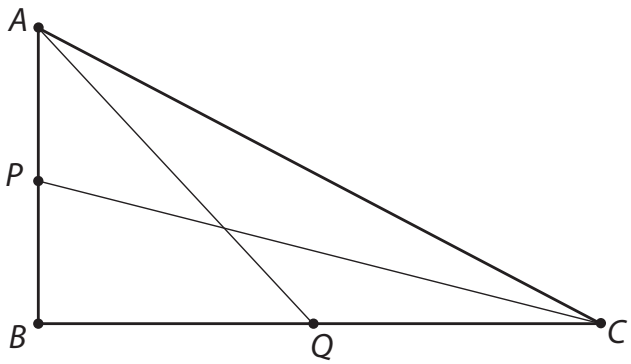
Question 2
What are the first two integers greater than 2000 with exactly 36 divisors each?

Question 3
A bag contains balls of various colours, with equal numbers of each colour. If two balls were drawn at random from the bag, let p be the probability that the two balls are the same colour. Michael now observes that if 64 balls of a new colour were added to the initial setup of the bag, and then two balls were drawn at random, the probability they are the same colour is once again p.
What is the largest number of balls that could have been in the bag initially?

Question 4
Let a, b and c be positive real numbers. What is the smallest value of

$$(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)?$$

Question 5
Triangle ABC is right angled at B, as shown below. P is a point on AB with $\angle BCP = \angle PCA$. Q is a point on BC with $\angle BAQ = \angle QAC$. If $AQ = 9$ and $CP = 8\sqrt{2}$, what is the length AC?



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