

2015 UQ/QAMT Problem Solving Competition - Year 11 & 12 Paper

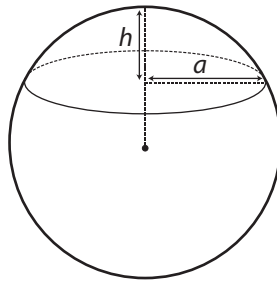
Two hours allowed. All questions have equal value. Non-CAS calculators may be used.

Question 1

What are the next three terms of the sequence 1309, 1310, 1311, 1885, 1886, 1887, ...?

Question 2

The volume of a spherical cap of height h and base radius a , as shown in the figure below, is $\frac{1}{6}\pi h(3a^2 + h^2)$.



Suppose that a cylindrical hole is bored straight through the centre of a solid sphere and that the length of the interior wall of the hole is 8 cm. What is the volume of the remaining part of the sphere?

Question 3

Take a circle and inscribe 4 non-overlapping equal sized circles A, B, C, D such that A is tangent to B , B is tangent to C , C is tangent to D and D is tangent to A such that each is also tangent to the large circle. In each of these 4 circles, inscribe 4 smaller tangent circles in exactly the same way, to get 16 small circles. Finally, inscribe a small circle C_{17} in the space around the centre of the large circle and tangent to each of A, B, C, D .

Which is larger, C_{17} or one of the 16 circles inside A, B, C, D ?

Question 4

An ant walks along a $5 \times 5 \times 5$ cube, from one corner to the diagonally opposite one, such that she only ever moves horizontally or vertically on the edges of the smaller cubes and that her distance from the ending corner never increases. How many different paths could she take?

Question 5

For which integer values of x is $x^3 - 6x^2 - 710x - 2015$ a prime number?

