THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

2013 UQ/QAMT Problem Solving Competition - Year 11 & 12 Paper

All questions have equal value. Non-CAS calculators may be used.

Question 1
Several regular polygons meet at a point in the plane leaving no gap. What is the largest number of sides any of the polygons can have?

Question 2
In terms of n, what is the coefficient of x^{n-2} in the expansion of $(x+1)(x+2)...(x+n)$?
Hint: the formula $1^2+2^2+\cdots+k^2=\frac{1}{6}k(k+1)(2k+1)$ *may be useful.*

Question 3
A triangle has internal angles α, β and γ at vertices A, B, C respectively. A point P inside the triangle is such that $\angle BAP=\angle CBP=\angle ACP=30^\circ$. What is the value of

$$\frac{1}{\tan\alpha}+\frac{1}{\tan\beta}+\frac{1}{\tan\gamma}?$$

Question 4
An eggsellent merchant sells eggs. One day a customer walked up and said, "I'll buy half of all your eggs, plus half an egg." The merchant sold the appropriate number of eggs. Soon after that another customer came along and said exactly the same thing, and again bought the appropriate number of eggs. Customers continued arriving and repeating the same request but the merchant lost count after the third customer.

Eventually the eggsellent merchant had sold all his eggs. The merchant only stocks prime quality merchandise, and so he is certain that the number of eggs he had originally was prime. What is the smallest number of customers he could have served?

Question 5
Four equilateral triangles with side length s are placed in a square of side length 1, as shown. What is the value of s?