2012 UQ/QAMT Problem Solving Competition - Year 11 & 12 Paper

All questions have equal value.

Question 1
Thirteen pirates are dividing a treasure of approximately 1000 doubloons between them. After dividing the coins into 13 equal heaps, there are 9 coins left over. To avoid an unfair division, 2 of the junior pirates are killed, and the loot redivided into 11 equal heaps. There are again 9 doubloons left over. One more pirate is killed, and now the coins divide exactly between the remaining pirates. Exactly how many coins are in the treasure?

Question 2
Suppose $0 < a < c$ and let
$$x = \frac{\sqrt{c+2a} - \sqrt{c+a}}{\sqrt{c} - \sqrt{c-a}}, \qquad y = \frac{\sqrt{c+2a} - \sqrt{c+a}}{\sqrt{c+a} - \sqrt{c}}, \qquad z = \frac{\sqrt{c} - \sqrt{c-a}}{\sqrt{c+2a} - \sqrt{c+a}}.$$
Which of these three numbers, x, y or z, is largest?

Question 3
Bob is playing a two-game chess match. Winning a game scores 2 points, and drawing a game scores 1 point. After the two games are played, the player with more points is declared the champion. If the two players are tied after two games, they continue playing until somebody wins a game (and hence the match).

During each game, Bob can choose to play *boldly* or *conservatively*. If he plays boldly, he has a 45% chance of winning and a 55% chance of losing that game. If he plays conservatively, he has a 90% chance of drawing and a 10% chance of losing. Assuming Bob follows an optimal strategy, what is the probability he wins the match?

Question 4
Three concentric circles of radii 1, 2, 3 respectively are drawn. An equilateral triangle is placed so that each of its vertices lies on a different circle, as shown. What is the side length of the triangle?
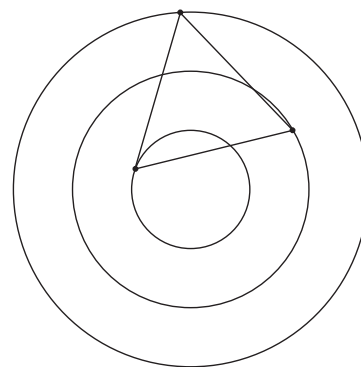
Question 5
Kato the wonder cat has 4 socks and 4 boots and she is puzzled by how many ways she can get dressed. Each sock and boot fits only one paw and each sock must be put on before the corresponding boot. In how many different ways can she put on her socks and boots?