THE UNIVERSITY
OF QUEENSLAND
AUSTRALIA

2011 UQ/QAMT Problem Solving Competition - Year 11 & 12 Paper

All questions have equal value.

Question 1
How many integers are of the form 2^n (where n is a natural number) such that after deleting the first digit of its decimal representation we again get a power of 2?

Question 2
The famous haberdasher's problem involves cutting an equilateral triangle so that the pieces can be rearranged into a square. One solution is given in the following diagram:

Suppose the triangle has area 1. What is the length of the longest cut in the triangle?

Question 3
Let a, b and c be distinct non-zero real numbers such that
$$a + \frac{1}{b} = b + \frac{1}{c} = c + \frac{1}{a}.$$
What is the value of $|abc|$?

Question 4
Draw n equally spaced marks between 0 and 1 on each of the x and y axes. Connect the first mark on the x-axis (closest to 0) to the last mark on the y axis (closest to 1), the second mark on the x axis to the second last on the y axis, and so on.

As n increases this process creates a curved boundary, C, as seen in this example diagram. Give an equation for this curve as $n \rightarrow \infty$.
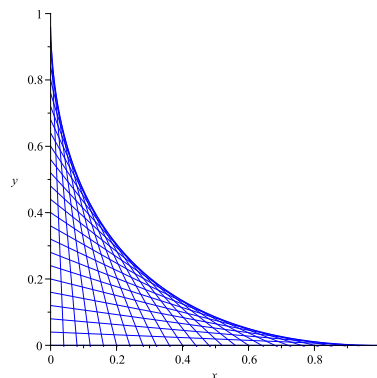
Question 5
Suppose r and s are positive integers with no common factor and
$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots + \frac{1}{1339} - \frac{1}{1340} = \frac{r}{s}.$$
Find a prime factor of r.

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