THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

2010 UQ/QAMT Problem Solving Competition - Year 11 & 12 Paper

All questions have equal value.

Question 1
The great pyramid is a square based pyramid whose base square has side length 1 and whose height is 3, in some system of units. What is the radius of the largest sphere that will fit entirely within the pyramid?

Question 2
What is the largest value of the constant c such that

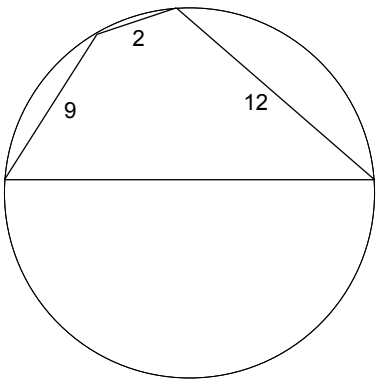
$$\frac{4-2cx}{c+4x} \geqslant \frac{1-x^2}{c+x}$$

for all values of $x \geqslant 0$?

Question 3
Let N be the (decimal) integer $\overbrace{11\cdots1}^{2010}$ (with 2010 digits). What is the 1000th digit after the decimal point of $\sqrt{N}$?

Question 4
Suppose f is a polynomial with integer coefficients such that $f(0) = f(1) = 0$ and $f(-1) = 30$. What is the smallest possible value of $|f(5)|$?

Question 5
A quadrilateral is drawn inside a circle such that its longest side is the diameter, and the remaining sides have length 2, 9 and 12. What is the diameter of the circle?



Question 6
If x is a real number, $\lfloor x \rfloor$ denotes the greatest integer that is less than or equal to x. For example $\lfloor \pi \rfloor = 3$. How many distinct elements are there in the set $\{\lfloor x^2/2010 \rfloor\}$ where $1 \leqslant x \leqslant 2010$?