2009 UQ/QAMT Problem Solving Competition - Year 11 & 12 Paper

All questions have equal value.

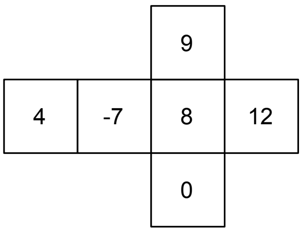
Question 1
What are the first four digits of 7^{2009}?

Question 2
Your semi-underground maths bunker has a 1 metre square window, with the bottom side exactly at ground level, parallel to the ground.

Suddenly a disc, 5 metres in diameter, rolls past just outside the window at velocity 1 m/s. For how long is at least part of your window obscured?

Question 3
Suppose you have positive real numbers $a_1, a_2, a_3, a_4, a_5, a_6, a_7$ not all equal, such that the average of the first k terms is equal to $\sqrt{a_1 a_k}$, for $k = 1, 2, \ldots, 7$. If $a_1 = 41$ what is the value of a_7?

Question 4
Four cubes are each formed from the pattern below and stacked on a table one on top of another. What is the greatest possible sum of the 17 visible numbers?



Question 5
In the Gregorian calendar years that are divisible by 4 are leap year unless they are divisible by 100 but not by 400. (So 2000 was a leap year while 1900 was not.) In 2009 Australia Day was on a Monday. What is the probability Australia Day will be on a Monday in a random year from the Gregorian calendar?

Question 6
Alice and Bob play a dice game. Initially there are three blank (6-sided, unbiased) dice. Alice writes the numbers from 1 to 18 on the 18 faces of the three dice, in any arrangement she chooses, with each number used once.

Bob then selects one die, and Alice selects one of the remaining two. Alice and Bob then roll their selected dice against each other simultaneously. They player who rolls the highest number wins. How can Alice number the dice so that she has a greater than 50% chance of winning?