THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

2008 UQ/QAMT Problem Solving Competition - Year 11 & 12 Paper

All questions have equal value.

Question 1
A positive integer n has first (decimal) digit 9. If this digit is moved to the end, the resulting integer is exactly $n/7$. What is the smallest n satisfying this condition?

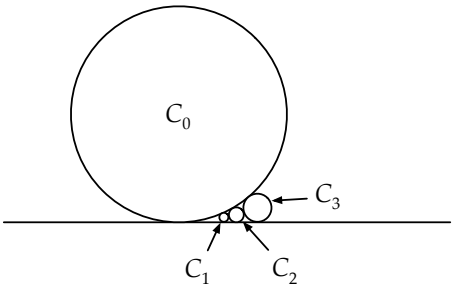
Question 2
A jail has n cells, labelled 1 to n. All are initially locked. The jailer makes n rounds of turning keys. In each round he begins at cell 1 and proceeds in order to cell n. In the first round he turns every key. In the second round he turns every second key. In the third round he turns every third key, and so on. In the last round he turns just the nth key. After this process, which cells are unlocked?

Question 3
A real-valued function f is defined on the integers, and satisfies:

(1) $f(0) = 1$
(2) $f(x + y + 1) + 3 = f(x + 1) + f(y + 1) + 2xy$

What is the value of $f(45) - f(4)$?

Question 4
C_0 is a circle of radius 10 metres tangent to the ground. C_1 is a circle of radius 1 millimetre, external to C_0 and tangent to both C_0 and the ground. For $n > 2$, C_n is a circle tangent to C_0, C_{n-1} and the ground. The diagram to the right shows the first few circles in this pattern (not to scale).



How many circles can be placed in this way (before they get too large to be tangential to C_0)?

Question 5
AB is the diameter of a semicircle. C is a point on the circle between A and B. D is another point on the circle between C and B, with distances $|AC| = 9$, $|CD| = 2$, $|DB| = 12$. What is the length of the diameter AB?

Question 6
Two people, A and B, agree to play the following game. A barrel is filled with a large number of balls. A proportion, p, of the balls are labelled A with the remaining proportion, $1 - p$, labelled B. The players take it in turns to draw a ball from the barrel. If it is labelled with their name they win and the game ends. Otherwise they replace their ball and the other player draws. If A goes first, what value of p would make the game fair (so that each player had a 50% chance of winning)?