

2007 UQ/QAMT Problem Solving Competition - Year 11 & 12 Paper

All questions have equal value.

Question 1

On an escalator moving at constant speed, a girl in a hurry walks up 9 steps as she travels from one floor to the next higher floor. A boy in an even greater hurry runs 25 steps up the same escalator and reaches the top in half the time that the girl took. Find the number of steps that the escalator has between the two floors.

Question 2

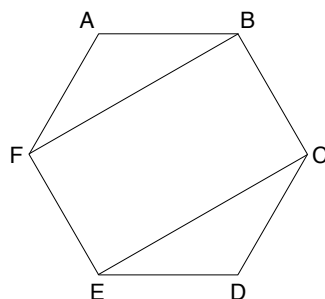
Find all triples (x, y, z) of positive integers that are solutions of the equation

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{5}{6}$$

Assume that $x \leq y \leq z$.

Question 3

The regular hexagon ABCDEF shown below has area 1. Find the area of the rectangle BCEF.



Question 4

Find all integers x such that $x^4 + 4$ is prime.

Question 5

Let $P_1 = (0, 1)$, $P_2 = (1, 1)$, $P_3 = (1, 0)$, $P_4 = (0, 0)$ be the vertices of the unit square. Let P_5 be the midpoint of P_1P_2 , P_6 be the midpoint of P_2P_3 , P_7 the midpoint of P_3P_4 , etc. Join the points P_1, P_2, P_3, \dots into a polygonal spiral. It is known that the spiral approaches a limiting point $P_\infty = \lim_{n \rightarrow \infty} P_n$. Find the coordinates of P_∞ .

Question 6

Let f be a function whose inputs and outputs are positive integers. Suppose f satisfies

$$f(n+1) > f(n), \quad f(f(n)) = 3n \quad \text{for all } n$$

Find $f(1278)$.