

2006 QAMT Problem-Solving Competition - Year 11 & 12 Paper

All questions have equal value.

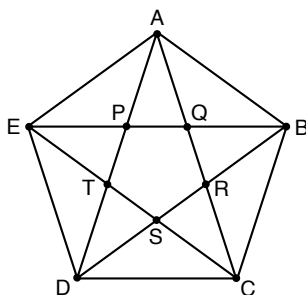
Question 1

Find all x such that

$$\sqrt{\frac{1 + \cos 4x}{2}} = \frac{1}{\sqrt{2}}.$$

Question 2

Connect all the vertices of a regular pentagon $ABCDE$ to each other with straight lines as shown. What is the ratio of the area of $ABCDE$ to the smaller pentagon $PQRST$? You may use the fact that $\cos(36^\circ) = \frac{\sqrt{5}+1}{4}$.



Question 3

Let $f = x^3 - px + q$ have roots α, β, γ . Express $(\alpha - \beta)^2(\beta - \gamma)^2(\gamma - \alpha)^2$ in terms of p and q .

Question 4

The *Koch snowflake* is constructed as follows. Start with an equilateral triangle of side length 1. Now divide each side into 3 equal parts and construct an equilateral triangle extending outwards on the middle section, and then delete its base. Repeatedly apply this subdivision step. The snowflake is the limiting shape obtained. What is its area?

Question 5

You have 6 coins, labelled 1 to 6 on one side and blank on the other. You toss all the coins into the air, let them fall, then make your best guess as to which coin is labelled 1. What is the probability you are correct?

Question 6

The hyperbolic functions are defined for all real numbers x as follows:

$$\sinh x = \frac{e^x - e^{-x}}{2}, \quad \cosh x = \frac{e^x + e^{-x}}{2}, \quad \tanh x = \frac{\sinh x}{\cosh x}.$$

Show that $\arcsin(\tanh x) = \arctan(\sinh x)$.