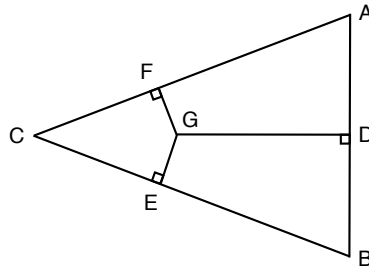# 2005 QAMT Problem-Solving Competition - Year 11 & 12 Paper

**Question 1**     If $x$ is a real number, $|x|$ denotes the distance of $x$ from 0. For example $|-3| = 3$, $|5| = 5$. Let R be the set of all points $(x, y)$ in the plane satisfying $|x| - |y| \leqslant 1$ and $|y| \leqslant 1$. Find the area of R.

2 marks

**Question 2**     In the diagram below, $FG + EG = DG$, $EG + DG = DA = 2EC = AF - FG$. Find the ratio $FC/EG$.



4 marks

**Question 3**     Five letters are written to five different addresses, and five matching envelopes with the addresses are prepared. How many ways can the letters be placed into the envelopes (one in each envelope) such that every letter is placed in a wrong envelope?

4 marks

**Question 4**     Alice, Bob and Cathy take turns (in that order) in rolling a six sided die. If Alice ever rolls a 1, 2 or 3 she wins. If Bob rolls a 4 or a 5 he wins, and Cathy wins if she rolls a 6. What is the probability that Cathy wins?

3 marks

**Question 5**     For which integer values of $x$ is $x^3 + 82x^2 - 34x - 2005$ a prime number?

3 marks

**Question 6**     Let $x$, $y$ and $z$ be positive real numbers. Show that

$$(x + y + z) \left( \frac{1}{x} + \frac{1}{y} + \frac{1}{z} \right) \geqslant 9.$$

4 marks