

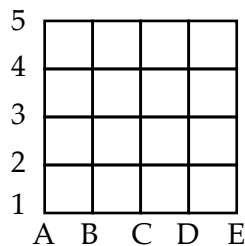
2004 QAMT Problem-Solving Competition - Year 11 & 12 Paper

Question 1 An integer $n > 1$ is said to be *abundant* if the sum of its factors (including 1 but excluding n) exceeds n . For example 12 is abundant since $1 + 2 + 3 + 4 + 6 = 16 > 12$. Find an odd abundant number less than 1000. 2 marks

Question 2 Three spherical melons, radius 9cm each, are placed on a flat table, each touching both the others. What is the radius of the largest orange that will sit on the table in the space between the 3 melons? 4 marks

Question 3 Three planets are orbiting a star in circular orbits. The first takes 12 years to make an entire orbit, the second 25 years and the third 32 years. Currently all three planets and the star are aligned along a straight line. How many years pass until this next happens? (The planets do not have to return to their starting positions, they just need to all line up.) 4 marks

Question 4 A taxi driver drives from the corner of 1st street and A avenue, to the corner of 5th street and E avenue in the diagram below. Each block counts as 1 unit, so the shortest possible path is 8 units. How many different paths are there of this length? 2 marks



Question 5 Show that $x^4 + y^4 = z^4 + 3$ has no integer solutions for x , y , and z . 4 marks

Question 6 Let $S_1 = 1$, $S_2 = 1 + 2$, $S_3 = 1 + 2 + 3$, etc. Evaluate

$$\frac{1}{S_1} + \frac{1}{S_2} + \dots + \frac{1}{S_{2004}}.$$

4 marks