THE UNIVERSITY OF QUEENSLAND
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Infinity

Maths magazine Number Winter 2005

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Elephants and trees

by Michael Bode

For many years, scientists believed that the natural state of an ecosystem was static: the "balance of nature" hypothesis. However it has recently been recognized that ecosystems can behave cyclically – even chaotically.

Elephants and trees behave typically as predator (elephants) and prey (trees) and their natural population levels can oscillate wildly. These oscillations in population levels can be viewed in the (T, E) plane (where the horizontal axis represents the number of trees, T, and the vertical axis the number of elephants, E) and modelled mathematically using a pair of coupled differential equations:

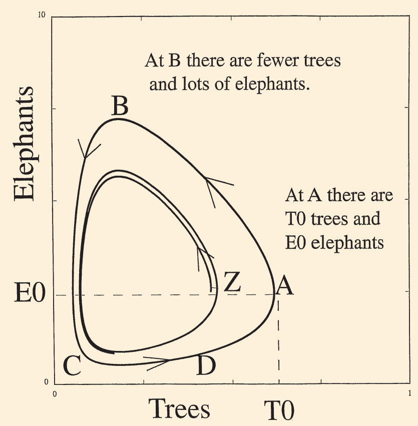
$$\frac{dT}{dt} = aT\left(1 - \frac{T}{K}\right) - \frac{bTE}{T+c} \text{ and } \frac{dE}{dt} = E\left(-d + \frac{eT}{T+c}\right),$$

where T is the abundance of trees, E is the population of elephants, and a, b, c, d and e are constants. Strikingly, the deterministic solution is a closed curve, implying a behavior that repeats itself, or behaves cyclically. For instance, starting at A, with the position indicating your initial number of elephants and trees, the ecosystem evolves along the solid line in the direction of the arrow (the figure below). The high elephant density causes over-grazing of the trees and the tree population decreases to B. The elephants then begin to starve, and their population levels crash rapidly to C. At this point, the trees are freed from grazing pressure, and so they recover to D. Then as more food becomes available, the elephants breed rapidly, increasing their numbers, and the cycle begins again.

As long as there are always some elephants and some trees the cycle continues. However if a catastrophe (e.g., a disease or a natural disaster) occurs just when one of the populations is low, a species can go extinct. Add to this the fact that elephant habitats are fragmented, and the elephant population, now compressed into small parks, is seriously threatened. But mathematical analysis tells us that by controlling the tree populations, we can decrease the risks faced by the elephants.

Mathematically, starting at Z rather than A means a smaller oscillation, at least for a couple of cycles. The elephant population doesn't grow as high. But, more importantly, it doesn't reach the dangerously low levels at C. Now starting at Z rather than A is simply a matter of reducing the number of trees. So removing trees at the right time and in the right area could mean that extinction is avoided.

continued inside...



At C there are very few trees or elephants!

Editorial

Hope you like the new look Infinity! You can visit our website at

www.maths.uq.edu.au/online/club_infinity

Year 12s will be choosing their university programs and courses for 2006 shortly. Don't forget to check out the UQ mathematics website

www.maths.uq.edu.au

to help with your selection.

Infinity Team:
Elizabeth Billington, Nicole Bordes, Diane Donovan, Cathy Holmes, Phil Isaacs and Barbara Maenhaut

Michiru Takizawa and Andrew Blinco

PROfiles

Three former UQ students are now working as Actuarial Analysts at Suncorp. They are training to become qualified actuaries while working full-time. The job of an actuary is primarily about predicting what will happen in the future. Stephen and Andrew work in General Insurance, calculating risks involved with insurance products, and Michiru works in Banking, providing advice to internal clients at Suncorp.

Stephen Long graduated with a BSc (Hons) in 2000 and a PhD in Combinatorial Mathematics and Chemistry in 2004. He enjoys problem solving and working with numbers. His main challenge at Suncorp is to present technical material in a manner that can be easily understood by a diverse audience. Suncorp provides him with many opportunities to experience a variety of jobs in the financial industry, and he would like to continue to work in this field. Stephen advises students to "study areas of mathematics that you enjoy, since the general skills learnt in any maths degree can be used in a wide range of jobs".

Michiru Takizawa graduated with a LLB in 1999, a BSc (Hons) in 2000, a Grad Cert IT in 2004, and expects to be awarded a PhD in Mathematical Physics in 2005. She says she's lucky to have found a job where she can continue her mathematical studies and also branch out into a new area of mathematics. Michiru hopes to become a qualified actuary and either stay in the finance industry or work in an actuarial education program. Her advice to students studying mathematics is to "choose what you enjoy the most. There are more opportunities out there for maths graduates than you may realise and the skills and knowledge you acquire will be highly valued by any employer".

Andrew Blinco graduated with a BSc (Hons) in 1998, a Dip Ed in 1997 and a PhD in Combinatorial Mathematics in 2003. He taught mathematics at a secondary school in London (UK) for a year and lectured at universities in China, the USA and Australia, before settling into his job at Suncorp in 2004. Andrew enjoys the many ways that mathematics can be used and at Suncorp he applies mathematics to real-life business problems. In his previous jobs, he particularly enjoyed researching new areas of mathematics because "you get a buzz knowing that you are solving problems that haven't been solved before". Andrew intends to stay at Suncorp and train to become a qualified actuary.

Elephants and trees

continued from front cover

Although the complete problem is more complicated than has been suggested here, analyzing these types of ecological problems mathematically leads to a more complete understanding of the dynamics at work and so to better management decisions. In 1900, the African elephant (*Loxodonta Africana*) ranged over 80% of West Africa. Their range has now been reduced to only 6%, mostly in small wildlife reserves scattered over the West African nations. The Ecology Center at The University of Queensland, of which I am a member, is currently using mathematics to make management decisions that will best preserve the remaining elephant herds, taking into account the costs and benefits of the many available options. As well as removing trees, these include translocation and culling of excess elephants, as well as sterilisation and administering the birth control pill.

Michael Bode is a research student with Professor Hugh Possingham in the Ecology Centre at The University of Queensland www.ecology.uq.edu.au

Crossnumber solution

(see back page)

1	3	■	6	4
2	5	2	■	4
1	■	3	6	1
■	3	3	0	■
3	6	■	1	2

Mayan arithmetic

The Ancient Maya from Mesoamerica developed an advanced system of arithmetic.

Our decimal system uses base 10; theirs used base 20 (referred to as the vigesimal system). In base 20, numbers are expressed in terms of powers of 20. For instance, the number 927 in base 10 is written as 267 in base 20, since 927 = 800 + 120 + 7 = **2** x 20^2 + **6** x 20^1 + **7** x 20^0.

However, the Maya did not use the Arabic notation with which we are familiar. Instead of the ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, that we use to represent a decimal number, the Maya used three symbols: the dot, the bar and the shell. The dot had a value of one, the bar a value of five and the shell a value of zero. The Mayan symbols for the numbers from 0 to 19 are shown in Figure 1.

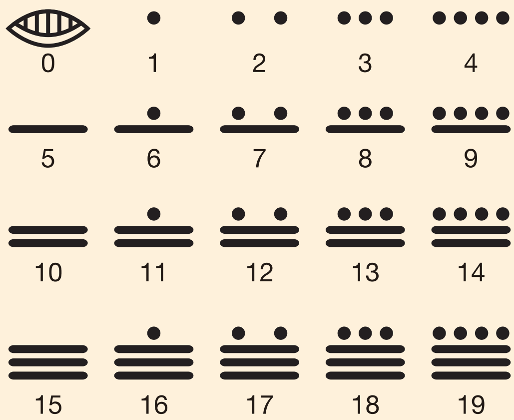


Figure 1: dot-and-bar notation for the numbers zero to nineteen (Sharer 1994) >

How did the Maya represent numbers greater than 19? They used their base 20 representation, but instead of writing the increasing powers of 20 from right to left as we do in the decimal system, they wrote them increasing from bottom to top. For example, the (decimal) number 927, which is **2** x 20^2 + **6** x 20^1 + **7** x 20^0, was written as shown in Figure 2.

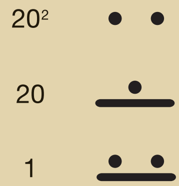


Figure 2: the (decimal) number 927 in Mayan arithmetic >

Addition of two Mayan numbers is performed by writing the two numbers (in column format) side-by-side and adding the corresponding rows. If any row sums to more than 19, then an extra one is carried into the next row up. As an example, let's add the decimal numbers 11,436 and 927 (see Figure 3). The decimal number 11,436 is written in base 20 as:
11436 = 8000 + 3200 + 220 + 16 = **1** x 20^3 + **8** x 20^2 + **11** x 20^1 + **16** x 20^0.

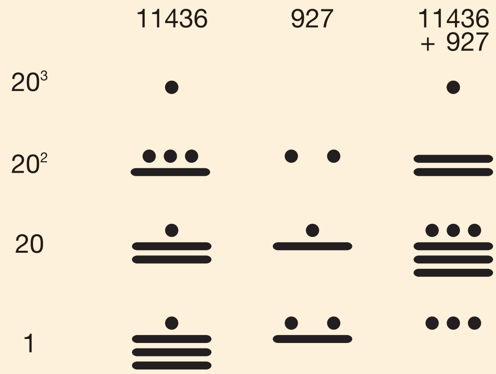


Figure 3: The addition of 11436 and 927 gives 1 x 20^3 + 10 x 20^2 + 18 x 20^1 + 3 x 20^0 >

Mayan merchants often used cocoa beans laid out on a flat surface to do their calculations.

Source: Robert J. Sharer, *The ancient Maya*, 5th edition, Stanford University Press, 1994

A puzzling dude

Henry Ernest Dudeney was born in the village of Mayfield, Sussex, England in 1857 and died in 1930. He was perhaps England's greatest creator of puzzles, or even the most creative who ever lived. Although he had only a very basic education, he studied mathematics and its history in his spare time.

During his lifetime he designed and published many mathematical puzzles and curiosities, many of which appeared in magazines. He was the author of several books, the two most famous being *Amusements in Mathematics* and *The Canterbury Puzzles*. His view on puzzles and the history of mathematics is summarised in his quote: "The history of mathematical puzzles entails nothing short of the actual story of the beginnings and development of exact thinking in man."

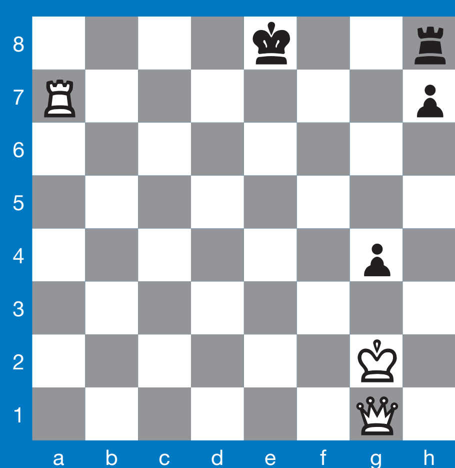
He often devised problems which interested other mathematicians, such as the following geometric novelty, known as "The Haberdasher's Problem":

Is it possible to dissect an equilateral triangle into four pieces that can be reassembled to form a square?

(The solution is on the back page.)

Several of his other problems can be found on the website www.puzzleworld.org/SlidingBlockPuzzles/classic.htm

Chess corner



Mate in two

To win a prize, send the solution to this problem to the same address for the competition on the back page.

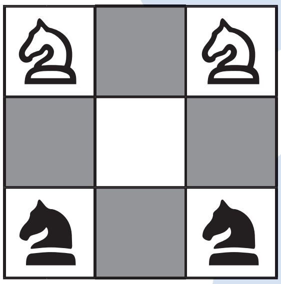
Galloping knights

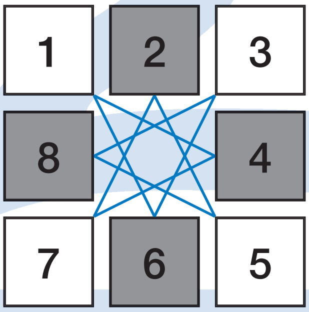
The question is what is the smallest number of knight's moves needed to exchange the positions of the white and black knights on the board shown below?

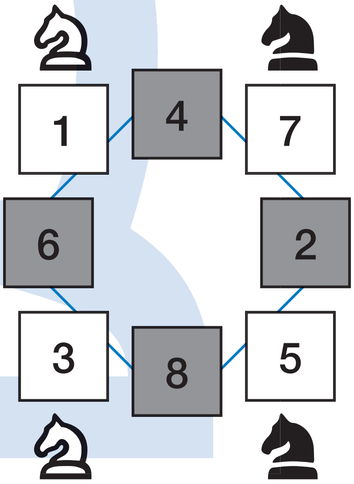
Don't forget that, in chess, knights move by first moving to an adjacent square and then moving two spaces in a perpendicular direction. So for instance the knight in the square labeled 1 can move to either the square labeled 4 or 6.

Thanks to Dudeney (see previous page), we have a novel solution to this puzzle. Notice that we cannot move a knight onto the central square, so it can be ignored. Next, label the squares 1 to 8. Join two squares by a line if you can move from one square to the other, using a knight's move. Now rearrange the squares to form a continuous circuit and notice that if you execute the moves 1→6, 3→8, 5→2, 7→4, followed by the moves 6→3, 8→5, 2→7, 4→1, you have rotated the knights through 90 degrees. Repeat this process and you have rotated the knights through 180 degrees, interchanging the black and white knights in 16 moves. The circuit diagram also verifies that this is a minimum number of moves, since the knights must all move in the same direction around the circuit

By representing the problem this way Dudeney provided a solution which uses a mathematical structure known as a graph or network: each square is called a vertex, each line is called an edge. Dudeney's problem is in part the problem of finding minimal paths between the corner vertices. ∞



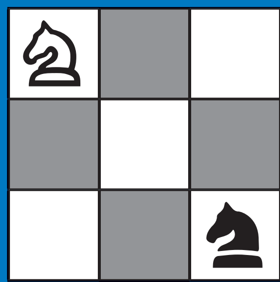




COMPETITION

Find the minimum number of knight's moves necessary to swap the positions of the black and white knights.

Send your answers to:
Infinity
UQ Mathematics
Level 6, Priestley Building,
The University of Queensland,
Brisbane Qld 4072
by 30 August 2005.
Answer will be posted on the Infinity website (www.maths.uq.edu.au/online/club_infinity) in early September.



Crossnumber

a number crossword

Each answer is a positive integer. You may find that some answers need to be completed before others can be determined!

ACROSS

1 The sixth prime number.
3 A perfect square which is also a sixth power.
5 The number of months in 21 years.
7 A perfect square
9 This number is divisible by 10 and by 11, but it's bigger than 110.
10 A triangular number (Triangular numbers are: **1**, 1+2=**3**, 1+2+3=**6**, 1+2+3+4=**10**, etc.).
11 You have four different magnetic letters to put on your fridge door. How many different two-letter nonsense words can you make from these four letters?

DOWN

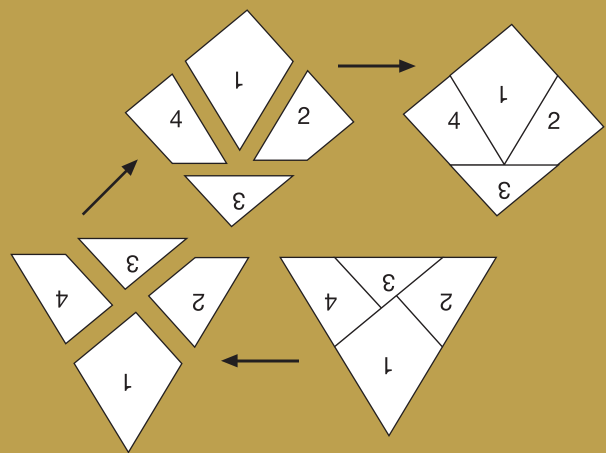
1 The square of 11.
2 The number of distinct groups of three colours you can pick from a group of seven colours, when the order doesn't matter (so red, white, blue is the same group as blue, red, white, for instance).
4 The sum of the cubes of the first six positive integers.
6 The Fibonacci numbers are 1, 1, 2, 3, 5, 8, 13,... where each number is the sum of the previous two. Starting with 1, 1, 2, 3,... as the first, second, third and fourth Fibonacci numbers, find the thirteenth Fibonacci number.
8 The smallest prime number greater than 600.
9 A perfect square. ∞

Solution on page • •

1	2	■	3	4
5		6	■	
	■	7	8	
■	9			■
10		■	11	

solutions

Dudeney: Haberdasher's Problem **(from previous page)**



Bisect two sides and divide the third side into three, roughly in the ratio .912 : 2 : 1.018. Draw the lines as indicated in the diagram below, with the lines meeting at right angles inside the triangle. Then rearrange the four pieces as shown.

Source: The MacTutor History of Mathematics archive.
www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Dudeney.html

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