Complex Analysis

MATH3401

Joseph Grotowski
Brief description of course content

This subject is an introduction to the theory of functions of a complex variable. In particular it will explore the consequences of differentiability on an open connected set, and demonstrate some of the applications. Specific topics include: Complex Numbers, Elementary Functions, Mapping by Elementary Functions, Analytic Functions, Integration, Conformal Mapping, Boundary Value Problems, Poisson Integral Formula, Power Series and Integration using Residues.

Lecturer and Course Coordinator:

Name: Joseph Grotowski
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Consultation hours (to be confirmed in the first week of class): Mon. 10-11, Tue 10-11.

Web page The course profile and course material can be found on the web at the following address:
http://www.maths.uq.edu.au/courses/MATH3401

This also contains up-to-date news about the course material and announcements for students. Please check this regularly during the semester.

Class contact hours:

<table>
<thead>
<tr>
<th>Lectures</th>
<th>Mon 9.00am</th>
<th>Tue 9.00am</th>
<th>Wed 11.00am</th>
<th>Weeks 1-13</th>
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<tbody>
<tr>
<td>Tutorial</td>
<td>Wed 3.00 pm</td>
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Regrettably tutors can only be paid to answer questions at tutorials, so please do not try to contact your tutor outside tutorials. Questions can be answered outside tutorial hours by visiting the lecturer during contact hours.

Assumed background:

Students are assumed to know the definition and basic properties of complex numbers and to be able to perform simple algebraic manipulations with them. Students are assumed to have undertaken introductory courses in Calculus and Multivariate Calculus (such as MATH1051 and MATH1052) and in Real Analysis (such as MATH2400).
Course goals

On completing this course students will:

- Be able to define a number of elementary functions on complex variables
- Know identities relating elementary functions
- Be able to define limits, continuity, and derivatives in the context of functions of complex variables
- Be able to determine limits and derivatives of certain functions of complex variables
- Be able to define analytic and entire functions and give some examples
- Be able to describe mappings by certain functions of complex variables
- Be able to define contour integrals and antiderivatives
- Understand and be able to clearly define and use the following: the Cauchy-Riemann equations and associated theorems, the Cauchy-Goursat theorem, the Cauchy integral formula and related theorems, Morera’s theorem, Liouville’s theorem, Rouche’s theorem, the Fundamental Theorem of Algebra, Taylor series, Laurent series, and the residue theorem
- Be able to recall proofs of some of the above results
- Be able to solve certain boundary value problems by conformal mapping and by applying integral formulae
- Be able to determine whether certain power series converge, and perform certain analytic and algebraic manipulations with power series
- Be able to evaluate certain types of integral using the residue theorem

Textbook and references

Perscribed Text:

You should plan on buying this book. Frequent references will be made to the book in class.

References:

Theory and Problems of Complex Variables (Schaum's Outline Series), Murray R. Spiegel, McGraw-Hill

Other references will be discussed in class.

Teaching and Learning Methods

Students should attend all the lectures. In truth most people cannot follow immediately all the details of a mathematics lecture; but try to get at least a broad overview of the material. Afterwards work through the material carefully, using lecture notes, and/or the corresponding section of the textbook. It is important to understand the examples discussed in lectures, and it is a good idea to make sure you can do the examples by yourself with the solution covered up. Of course this does not mean memorizing the solution: rather, it is a check that you understand the key steps involved.
A script will not be provided, nor will one be posted on the web-page. If you miss a lecture, obtain lecture notes from one of your fellow students.

There will be various hand-outs throughout the semester: these will be posted on the web-page, along with assignments, and model solutions.

Tutorials will provide students with an opportunity to ask questions on lecture material, to receive hints and pose questions about current assignments, and to receive comprehensive solutions to assignments you have handed up. You can get help with any difficulties from your tutor (or another student) at the tutorials. You may also work on assignment questions with other students. **However, make sure you understand the answer to the problem and write up your own solution by yourself.** When the assignments are returned, check with your tutor and the model solution on the web to see where you made errors. Students who hand in all the problems and make sure they understand any errors they made are very likely to pass the subject.

**COURSE SCHEDULE:**

The semester consists of 37 lectures (13 weeks, 3 lectures per week: 25 April, 1 May are public holidays). The first lecture will be an introduction, the final lecture will be revision. One lecture in week 6 will be devoted to the mid-term.

The following is a draft of the lecture schedule for the course. Students should read the assigned readings *prior* to each lecture. The optional readings are not directly examinable and should be read only by students who feel they have comfortably mastered the material in the assigned readings. Students will be advised of any changes to the assigned readings as soon as possible.

**1. Overview and Revision**

*Optional reading:* Sections 1-9.

**1. Elementary Functions and Mappings**

**2. Mappings and Functions**

*Pre-reading:* Sections 11, 12(exclude examples), 83, 84, 85.

**3. Linear Fractional Transformations**

*Pre-reading:* (Sections 86, 87, 88 (exclude example 3)).

**4. Exponentials and Logarithms**

*Pre-reading:* Sections 13, 28, 29, 31, 88(example 3 only).

**5. Complex Exponents**

*Pre-reading:* Sections 32, 12(Examples only), 90(exclude Example 3), 91.

**6. Trigonometric and Hyperbolic mappings**

*Pre-reading:* Sections 33, 34, 35, 89, 90 (Example 3 only).
II. Calculus of Analytic Functions

7. Limits
Pre-reading: Sections 10, 14-16.

8. Continuity and Differentiation
Pre-reading: Sections 17-19.

9. Analytic Functions and the Cauchy-Riemann Equations
Pre-reading: Sections 20-24.
Optional reading: Sections 101-103.

10. Integration of Complex-Valued Functions
Pre-reading: Sections 36-38.

11. Contour Integration
Pre-reading: Sections 39, 40.

12. Antiderivatives and the Cauchy-Goursat Theorem
Pre-reading: (7th Ed.) Sections 42-44.

13. Proof of the Cauchy-Goursat Theorem
Pre-reading: (7th Ed.) Sections 41, 45, 46.

14. Cauchy Integral Formula, Morera’s Theorem
Pre-reading: (7th Ed.) Sections 47, 48.

15. Liouville’s Theorem, Fundamental Theorem of Algebra, and review for mid-semester exam
Pre-reading: (7th Ed.) Sections 47, 48.

16. Mid-semester Examination
Based on first 15 assigned readings and lectures.

III. Conformal Mapping and Boundary Value Problems

17. Conformal Mapping
Pre-reading: Sections 94-96.

18. Harmonic Functions
Pre-reading: Sections 25, 97, 98.


22. Temperatures and Electrostatic Potentials
Pre-reading: Sections 102-105.

21. Fluid Flow
Pre-reading: Sections 106-108.
22. Schwarz-Christoffel Transformation  
*Pre-reading:* Sections 109, 110.

23. Examples of Schwarz-Christoffel Transformation  
*Pre-reading:* Sections 111, 112.

24. Applications of Schwarz-Christoffel Transformation  
*Pre-reading:* Sections 113-115.

25. Poisson Integral Formula  
*Pre-reading:* Sections 116-118.

26. More Integral Formula  
*Pre-reading:* Sections 119-121.

IV. Series

27. Power Series  
*Pre-reading:* Sections 51, 52, 57, 58.

28. Taylor Series  
*Pre-reading:* Sections 53, 54.

29. Laurent Series  
*Pre-reading:* Sections 55, 56.

30. Manipulations with Power Series  
*Pre-reading:* Sections 59-61.

V. Integration using Residues

31. Residues and Poles  
*Pre-reading:* Sections 62-65.

32. Integration using Residues  
*Pre-reading:* Sections 66-69.  
*Optional reading:* Section 70.

33. Improper Integrals  
*Pre-reading:* Sections 71-73.

34. Jordan’s Lemma and Indented Paths  
*Pre-reading:* Sections 74-77.

35. Rouche’s Theorem  
*Pre-reading:* Sections 79-80.

36. Some Applications  
*Pre-reading:* Sections 78, 81, 82.
37. Review

ASSESSMENT

Required assessment tasks:

Students should do the bi-weekly assignments to be handed out in weeks 2--12. These are handed in at before the Wednesday lecture, and will be returned in tutorials the following week. No responsibility will be taken for assignments which are not stapled, or are where student is not adequately identified. Solutions will be discussed in tutorials, so late submission will not be permitted. Students may receive help on assignments, but they must write out their solution themselves in their own words. Students whose solutions are copied from other students, the web, or from books without referencing the original source are violating the University’s policy on plagiarism (given below). Students whose work does not display an independent approach will not receive credit for the assignment. The assignments count 25% towards the final mark. The worst single assignment mark will be dropped in determining your mark for assignments, and the remaining assignments will be equally weighted.

Tentative due dates for assignments:
15/3, 29/3, 12/4, 3/5, 17/5, 31/5.

Students should check that assignment marks are correctly entered on the maths system:


No discussions about incorrect/missing marks will be entertained more than three weeks after the due date of the given assignment.

There will be a mid-semester exam to be held in class, at a date to be confirmed on the web and in lectures as soon as possible. The mid-semester exam counts 25% towards the final mark.

Date for the mid-semester exam: Tuesday, 4 April 2006.

There will be a final exam in the examination period at the end of first semester. This counts 50% towards the final mark.

Hand-held calculators will be allowed on the mid-semester or final exams, but not programmable, graphing or ASCII calculators. This policy will be strictly enforced.

Students will be permitted one page (single sided) of hand-written notes for the mid-term, and one page (double sided) for the final exam. These notes must be written and signed by the student. No printed matter, mechanical copies or notes written by others will be permitted. To obtain the final grade, the marks will be weighted as described above and added to give a final mark out of 100. Students will receive a grade from 1 to 7 if their mark is above the following cut offs. Note that grades of 4, 5, 6 and 7 are passing grades, and that 1, 2 and 3 are failing grades.
Assessment criteria

To earn a Grade of 7 (Pass), a student must demonstrate an excellent understanding of the course material. This includes clear expression of nearly all their deductions and explanations, the use of appropriate and efficient mathematical techniques and accurate answers to nearly all questions and tasks with appropriate justification. They will be able to apply mathematical techniques to completely solve both theoretical and practical problems.

To earn a Grade of 6 (Pass), a student must demonstrate a comprehensive understanding of the course material. This includes clear expression of most of their deductions and explanations, the general use of appropriate and efficient mathematical techniques and accurate answers to most questions and tasks with appropriate justification. They will be able to apply mathematical techniques to partially solve both theoretical and practical problems.

To earn Grade of 5 (Pass), a student must demonstrate an adequate understanding of the course material. This includes clear expression of some of their deductions and explanations, the use of appropriate and efficient mathematical techniques in some situations and accurate answers to some questions and tasks with appropriate justification. They will be able to apply mathematical techniques to solve fundamental problems.

To earn a Grade of 4 (Pass), a student must demonstrate an understanding of the basic concepts in the course material. This includes occasionally expressing their deductions and explanations clearly, the occasional use of appropriate and efficient mathematical techniques and accurate answers to a few questions and tasks with appropriate justification. They will have demonstrated knowledge of techniques used to solve problems and applied this knowledge in some cases.

To earn a Grade of 3 (Fail), a student must demonstrate some knowledge of the basic concepts in the course material. This includes occasional expression of their deductions and explanations, the use of a few appropriate and efficient mathematical techniques and attempts to answer a few questions and tasks accurately and with appropriate justification. They will have demonstrated knowledge of techniques used to solve problems. Falls short of satisfying all basic requirements for a pass.

To earn a Grade of 2 (Fail), a student must demonstrate some knowledge of the basic concepts in the course material. This includes attempts at expressing their deductions and explanations and attempts to answer a few questions accurately. Falls short of satisfying all basic requirements for a pass.

To earn a grade of 1 (Fail), a student will earn show a poor knowledge of the basic concepts in the course material. This includes attempts at answering some questions but showing an extremely poor understanding of the key concepts. Falls short of satisfying all basic requirements for a pass.
Assessment policy

As solutions to assignments are distributed promptly, credit cannot be given for late assignments. Students who miss assignments through bereavement or illness should document their problems and discuss this with the lecturer. They may be given an average mark for missed assignments.

Students who miss the mid-semester exam through bereavement or illness should document their problems and discuss this with a lecturer. A special mid-semester examination may be awarded. Allowance cannot be made for reasons such as sporting or social commitments, or overwork in other courses.

Tentative date for the special mid-semester exam: Saturday, 6 May.

Students should be familiar with the assessment rules in their degrees as well as general university policy such as found in the General Award Rules. These are all set out on the Program and Course Information page on the UQ website http://www.uq.edu.au/student/courses/.

Plagiarism:

Below is the University’s definition of plagiarism.

Plagiarism is the action or practice of taking and using as one’s own the thoughts or writings of another (without acknowledgement). The following practices constitute acts of plagiarism and are a major infringement of the University’s academic values:

(a) where paragraphs, sentences, a single sentence or significant part of a sentence which are copied directly, are not enclosed in quotation marks and appropriately footnoted;

(b) where direct quotations are not used, but are paraphrased or summarised, and the source of the material is not acknowledged either by footnoting or other simple reference within the text of the paper;

(c) where an idea which appears elsewhere in print, film or electronic medium is used or developed without reference being made to the author or the source of that idea.

When a student knowingly plagiarises someone’s work, there is intent to gain an advantage and this may constitute misconduct.

Students are encouraged to study together and to discuss ideas, but this should not result in students handing in the same or similar assessment work. Do not allow another student to copy your work. While students may discuss approaches to tackling a tutorial problem, care must be taken to submit individual and different answers to the problem. Submitting the same or largely similar answers to an assignment or tutorial problem may constitute misconduct.
For more information on the University policy on plagiarism, please refer to http://www.uq.edu.au/hupp/contents/view.asp?s1=3&s2=40&s3=12

Supplementary examinations

In some programs, a supplementary examination may be awarded in one course to students who obtain a grade of 2 or 3 in the final semester of their program and require this course to finish their degree. You should check the rules for your degree program for information on the possible award of supplementary examinations. Note that recent changes to university Policy concerning the award of a grade of 3 could effect this. Applications for supplementary examinations must be made to the Director of Studies in the Faculty.

Special examinations

If a student is unable to sit a scheduled examination for medical or other adverse reasons, she/he can and should apply for a special examination. Applications made on medical grounds should be accompanied by a medical certificate; those on other grounds must be supported by a personal declaration stating the facts on which the application relies.

Applications for special examinations for central and end-of-semester exams must be made through the Student Centre. Applications for special examinations in school exams are made to the course coordinator.

More information on the University’s assessment policy may be found http://www.uq.edu.au/hupp/contents/view.asp?s1=3&s2=30&s3=5

EPSA Faculty policy on the award of special and supplementary exams may be found at http://www.epsa.uq.edu.au/index.html?id=9329&pid=7564

Feedback on assessment:

You may request feedback on assessment in this course progressively throughout the semester from the course coordinator. Feedback on assessment may include discussion, written comments on work, model answers, lists of common mistakes and the like. (http://www.uq.edu.au/hupp/contents/view.asp?s1=3&s2=30&s3=6)

Students may peruse examinations scripts and obtain feedback on performance in a final examination provided that the request is made within six months of the release of final course results. After a period of six months following the release of results, examination scripts may be destroyed.

Information on the University’s policy on access to feedback on assessment may be found at http://www.uq.edu.au/hupp/contents/view.asp?s1=3&s2=30&s3=5

EPSA Faculty policy on feedback and re-marking may be found at http://www.epsa.uq.edu.au/index.html?id=7674&pid=7564
Library contact:
The liaison librarian for Earth Sciences/Maths/Physics is located in the Physical Sciences and Engineering Library in the Hawken Building and may be consulted for assistance in the course:

Maths: Leith Woodall
Email: lwoodall@library.uq.edu.au
Extension: 52367

Students with disabilities:
Any student with a disability who may require alternative academic arrangements in the course is encouraged to seek advice at the commencement of the semester from a Disability Adviser at Student Support Services.

Assistance for Students:
Students with English language difficulties should contact the course coordinator or tutors for the course. Students with English language difficulties who require development of their English skills should contact the Institute for Continuing and TESOL Education on extension 56565.

The Learning Assistance Unit located in the Relaxation Block in Student Support Services. You may consult learning advisers in the unit to provide assistance with study skills, writing assignments and the like. Individual sessions are available. Student Support Services also offers workshops to assist students. For more information, phone 51704 or on the web http://www.sss.uq.edu.au/index.html.

Student Liaison Officer:
The School of Physical Sciences has a Student Liaison Officer as an independent source of advice to assist students with resolving academic difficulties.

The Student Liaison officer during 1st Semester 2006 is:
Prof. Peter Adams,
Room 67-547, Priestley Building
Phone number: 336-532760
Email: pa@maths.uq.edu.au

Graduate Attributes:
The following graduate attributes will be developed in the course –
In-Depth Knowledge of the Field of Study

- A comprehensive and well-founded knowledge of the field of study:- through solving problems.
- An understanding of how other disciplines relate to the field of study:- through applying the mathematical techniques of the course to simple problems from other disciplines.
- An international perspective on the field of study:- through using internationally accepted standards of mathematical rigour and notation.

Effective Communication

- The ability to collect, analyse, and organise information and ideas, and to convey those ideas clearly and fluently, in both written and spoken forms:- through tutorial participation and assignment presentation, and through library usage.
- The ability to interact effectively with others in order to work towards a common outcome:- through cooperative learning strategies in practicals.
- The ability to select and use the appropriate level, style and means of communication:- through assignments and practicals.
- The ability to engage effectively and appropriately with information and communication technologies:- through practical use of computers.

Independence and Creativity

- The ability to work and learn independently.
- The ability to generate ideas and adapt innovatively to changing environments.
- The ability to identify problems, create solutions, innovate and improve current practices.

Critical Judgement

- The ability to define and analyse problems
- The ability to apply critical reasoning to issues through independent thought and informed judgement
- The ability to evaluate opinions, make decisions and to reflect critically on the justifications for decisions.

Ethical and Social Understanding

- An understanding of social and civic responsibility
- An appreciation of the philosophical and social contexts of a discipline
- A knowledge and respect of ethics and ethical standards in relation to a major area of study:- through the experience of a discipline where the concepts of right and wrong are supported by universal and absolute standards.
• *A knowledge of other cultures and times and an appreciation of cultural diversity:* through tutorial participation in a subject taken by students with diverse backgrounds and interests, and through an appreciation of the historical concept of the mathematical techniques used.

For more information on the University policy on development of graduate attributes in courses, refer to the web