1. Solve the following SDEs by any means. Some suggested methods are listed.

   (a) \( dR_t = c(\mu - R_t)dt + \sigma dW_t \)
       (general method for linear SDEs)

   (b) \( dX_t = X_t^3 + X_t^2 dW_t \)
       (change of time, convert to a Statonovich SDE)

   (c) \( dX_t = \left(t - \frac{1}{8W_t^2}\right) X_t dt + \frac{X_t}{2W_t} dW_t \)
       (Ito’s formula)

   (d) \( dX_t = \frac{1}{X_t} dt + \alpha X_t dW_t \)
       (change of measure to eliminate drift)

   (e) \( dY_t = \frac{b-Y_t}{t} dt + dW_t \)

2. In the following, either find an arbitrage opportunity or show that none exist:

   \[
   \begin{align*}
   dB_t &= 0 \\
   dS_t^{(1)} &= 2dt + dW_t^{(1)} \\
   dS_t^{(2)} &= -dt + dW_t^{(1)} + dW_t^{(2)}
   \end{align*}
   \]

3. In the following, show that the market is complete or find an \( \mathcal{F}_T \) measurable claim which is not attainable.

   \[
   \begin{align*}
   dB_t &= 0 \\
   dS_t^{(1)} &= dt + dW_t^{(1)} + dW_t^{(2)} \\
   dS_t^{(2)} &= 2dt + dW_t^{(1)} - dW_t^{(2)} \\
   dS_t^{(3)} &= -2dt - dW_t^{(1)} + dW_t^{(2)}
   \end{align*}
   \]

4. Suppose \( r, \mu \) and \( \sigma \) are all positive constants and the market is modeled by

   \[
   \begin{align*}
   dB_t &= rB_t dt, B_0 = 1 \\
   dS_t &= (\mu - S_t) dt + \sigma dW_t, S_0 = 1
   \end{align*}
   \]
This is the mean-reverting O-U process. (a) Find the time $t$ price of the European-style claim which pays $S_T$ at maturity time $T$. (b) Find the replicating portfolio for this claim.