MATH2300  
Graph Theory Problem Sheet 2

1. Let $F$ be a forest with $p$ vertices, $q$ edges and $k$ components. Show that $p = q + k$.

2. Show that not all graphs with $p$ vertices and $p - 1$ edges are trees.

3. Suppose that $T$ is a tree with $p$ vertices all of which are of degree 1 or 3. Show that $T$ has exactly $(p - 2)/2$ vertices of degree 3.

4. Let $T$ be a tree with 21 vertices having degree set $\{1, 3, 5, 6\}$. If $T$ has 15 vertices of degree 1 and one vertex of degree 6, how many vertices of degree 5 does $T$ have?

5. Prove that if $d_1, d_2, \ldots, d_p$ is the degree sequence of a tree, then $d_1+1, d_2, d_3, \ldots, d_p, 1$ is the degree sequence of a tree.

6. A connected graph $G$ has degree sequence 8, 8, 7, 7, 6, 6, 6, 5, 4, 4, 3. How many edges must be removed from $G$ so that the resulting graph is a spanning tree of $G$?

7. For the graphs shown below, find the depth-first search forest and the breadth-first search forest.

8. Determine all graphs $G$ of order $p \geq 4$ such that the subgraph induced by every three vertices of $G$ is a tree — or show that no such graphs $G$ exist.

Continued over page
9. Use Kruskal’s algorithm to find a minimum spanning tree in the weighted graph shown below.

10. Repeat the previous question using Prim’s algorithm.

End of Problem Sheet 2