MATH2300
Graph Theory Assignment 1
Due Monday October 4th at 5pm (Hand in at a lecture, tute, or to room 67-448)

1. A graph $G$ is self-complementary if it is isomorphic to its complement $G^c$.

   (a) Draw a self-complementary graph of order 5.
   (b) Prove that a self-complementary graph must have order $p \equiv 0$ or 1 (mod 4).

2. (a) A graph has 14 vertices and 25 edges and every vertex has degree 3 or 5. How many vertices of degree 3 does the graph have?
   (b) A graph has 7 vertices, 10 edges, six vertices of degree $a$ and one of degree $b$. What is $b$?

3. Determine whether the following degree sequences are graphical; if a sequence is graphical, draw a graph having that degree sequence; if a sequence is not graphical, briefly explain why.

   $S_1 : 8, 5, 5, 4, 3, 2, 2, 2.$
   $S_2 : 7, 7, 4, 3, 3, 3, 2, 2, 1.$
   $S_3 : 6, 6, 4, 3, 3, 3, 1.$

4. Find a counterexample to each of the following statements:

   (a) If $G$ is a connected graph that contains only even vertices, then $G$ contains no cut-vertices.
   (b) If $G$ is a connected graph such that every vertex of $G$ lies on a cycle of $G$, then $G$ contains no cut-vertices.
   (c) If $G$ is a connected graph with a cut-vertex, then $G$ contains a bridge.
   (d) If $G$ is a connected graph with a bridge, then $G$ contains a cut-vertex. (Yes, there is a counterexample to this one!)

5. (a) Determine $\lambda(K_{m,n})$ and $\kappa(K_{m,n})$, where $1 \leq m \leq n$.
   (b) Determine $\lambda(K_{1,m,n})$ and $\kappa(K_{1,m,n})$, where $1 \leq m \leq n$.
   (c) If $G$ is a graph of order $p$ (with $p \geq 2$), and if $\delta(G) \geq (p - 1)/2$, prove that $G$ is connected.

6. (a) Show that any graph $G$ contains a path of length $\delta(G)$.
   (b) Draw an eulerian graph with an even number of vertices and an odd number of edges, or prove that this is impossible.

   End of Assignment