Complete all of the following problems and hand in your solutions to your tutor by the due date and time. Make sure that your name and student number are on each sheet of your answers. Solutions to all the problems will be distributed later. Completing 5 out of the 6 assignments is compulsory and each of the five assignments will contribute 4% towards your final grade. Late assignments will not be accepted unless you have a sufficient documented reason, such as illness.

1. (a) Show that \((\mathbb{R}, +, \times)\) is a field, where + and \(\times\) denote regular addition and multiplication, respectively.

   (b) Explain why \((\mathbb{Z}_8, \oplus, \odot)\) is not a field, where \(\oplus\) denotes addition modulo 8 and \(\odot\) denotes multiplication modulo 8.

2. Complete the following problems from the textbook:

   - Second edition, Section 6.1: pages 279-280: Questions 8bc, 11, 18
   - Second edition, Section 6.4, pages 320-322: Questions 13bc, 16

   or

   - Third edition, Section 6.1: pages 304-306: Questions 14bc, 22, 29
   - Third edition, Section 6.2, pages 318-320: Questions 5, 15 (but use 20 combinations, not 30), 30, 37
   - Third edition, Section 6.3, pages 330-333: Questions 22, 27 (but change the numbers to 40 people all up, and the numbers reporting relief from the drugs to 23, 18, 31, 11, 19, 14, 37).
   - Third edition, Section 6.4, pages 347-349: Questions 13bc, 16

3. Find simplified expressions for each of the following.

   (a) \(\left( \frac{y + 1}{y - 2} \right)\)

   (b) \(\binom{10}{4} + \binom{9}{4} + \binom{9}{5}\)

4. (a) Use the Binomial Theorem to expand and simplify \((2x - y)^5\).

   (b) Find the coefficient of \(p^3q^5\) in the expansion of \((p - q)^8\).
5. Nine lazy MATH1061 students develop a scheme for doing their assignment. They decide that a team of four people can do the work, and the rest of the students can play computer games in the lab, or eat. In each part (a) to (f), the extra conditions apply only in that part. In answering each question, evaluate your answers in full, and show working.

(a) How many different teams of size 4 can be selected from the 9 people?

(b) Two of the students are keen on each other and are inseparable, so one will not be on the team without the other. Thus they must either both be on the team, or neither on the team. How many distinct selections of 4 people can be made?

(c) Lester is so attractive that girls cannot concentrate when they are with him. If four of the nine are girls, and no girl can be on the team if Lester is on the team, then how many distinct selections of 4 people can be made?

(d) If Darryn and Bob are both on the team, then they will spend their whole time telling rude jokes, and the team will fail. If at most one of Darryn and Bob can be on the team, then how many distinct selections of 4 people can be made?

(e) Annette loves maths, and can’t think of anything better to do than maths. What is the probability that she will be part of the team (assuming selection is random)?

(f) Two students love math, four students hate math and three students don’t care either way. How many distinct teams contain exactly one person who loves math, two who hate math and one who doesn’t care either way?

(g) Five of the students are male, and four are female. If selection is random, what is the probability that the team will contain at least one male, and at least one female?