Complete all of the following problems and hand in your solutions to your tutor by the due date and time. Make sure that your name and student number are on each sheet of your answers. Solutions to all the problems will be distributed later. Completing 5 out of the 6 assignments is compulsory and each of the five assignments will contribute 4% towards your final grade. Late assignments will not be accepted unless you have a sufficient documented reason, such as illness.

1. Do each of the following.
   (a) Evaluate $\sum_{i=1}^{n} \frac{1}{2^i}$ for $n = 4$. What would you expect to happen to the sum as $n$ gets very large?
   (b) Write $(1^2 - 1) + (2^3 - 1) + (3^4 - 1) + (4^5 - 1)$ in summation notation.
   (c) Evaluate $\prod_{j=0}^{5} (-1)^j$.
   (d) Write $(1 - t^2) \cdot (2 - t^2) \cdot (3 - t^2) \cdot (4 - t^2)$ in product notation.
   (e) Evaluate $\frac{7!}{5!2!}$.

2. After any football match, three things happen:
   (i) every player in the match shakes hands with every other player;
   (ii) the captain of the winning team says to the interviewer “Full credit to the losing team”; and
   (iii) the players all have a shower together.

   (a) Assume there are $n$ players on the field. Explain (in english) why the total number of handshakes is given by
   $$1 + 2 + 3 + \ldots + (n - 3) + (n - 2) + (n - 1).$$

   (b) Write the expression from part (a) in summation notation.

   (c) Use mathematical induction to prove that, provided that there are at least two people on the field, the number of handshakes is
   $$\frac{n(n-1)}{2}.$$

   (d) The formula from part (c) appears surprisingly often in real-life. Suppose that there are $n$ teams in a sporting competition. Every team $A$ plays every other team $B$ twice, once at the home ground for team $A$, and the other time at the home ground for team $B$.

   (i) Using the result from part (c), explain (in english) why, in a competition with $n$ teams, there will be $n(n - 1)$ games played in a whole season.

   (ii) If $n$ is even and each team plays one game per week, how many weeks will there be in a whole season?
(iii) If \( n \) is odd, and each team plays one game per week with one team having a bye each week (so one team misses playing each week), how many **weeks** will there be in a whole season?

3. Complete the following problems from the textbook:

   - 2nd edition: Section 4.2, pages 204-205: Questions 10, 15 (see hint below for question 15)
   - 2nd edition: Section 4.3, pages 210-211: Questions 20b, 24
   or
   - 3rd edition: Section 4.2, page 226: Questions 11, 16 (see hint below for question 16)
   - 3rd edition: Section 4.3, pages 233-234: Questions 23b, 27

   (Hint for Question 15/16: You are asked to prove that \( \prod_{i=2}^{n} \left( 1 - \frac{1}{i^2} \right) = \frac{n+1}{2n} \). You can use the fact that \( \left( 1 - \frac{1}{(k+1)^2} \right) = \frac{k(k+2)}{(k+1)^2} \).

4. Complete the following problems from the textbook:

   or
   - 3rd edition: Section 5.1, pages 267–268: Questions 8cdeghj, 14cd, 15bcdef

5. Use propositions and a truth table to prove that for arbitrary sets \( A \) and \( B \),

   \[ \text{if } A \subseteq B, \text{ then } (A \cap B) \subseteq B. \]

6. Use Venn diagrams to illustrate that for arbitrary sets \( A, B \) and \( C \),

   \[ (A - B) \cap (C - B) = (A \cap C) - B. \]

7. Let \( A \) be the empty set \( \emptyset \), let \( B \) be the set containing \( A \), and let \( C = \{ B \} \).

   (a) Carefully write down each of \( A, B \) and \( C \).

   (b) State whether each of the following statements is true or false.

      (i) The empty set is a member of \( A \).
      (ii) The empty set is a member of \( B \).
      (iii) The empty set is a member of \( C \).
      (iv) The empty set is a subset of \( A \).
      (v) The empty set is a subset of \( B \).
      (vi) The empty set is a subset of \( C \).
      (vii) \( A \) is a subset of \( B \).
      (viii) \( A \) is a subset of \( C \).
      (ix) \( B \) is a subset of \( C \).
      (x) \( A \) is an element of \( B \).
      (xi) \( A \) is an element of \( C \).
      (xii) \( B \) is an element of \( C \).

   (c) Let \( D \) be the powerset of \( A \) and \( E \) be the powerset of \( B \). Write down \( D \) and \( E \).

   (d) State whether each of the following statements is true or false.

      (i) The empty set is a member of \( D \).
      (ii) The empty set is a member of \( E \).
      (iii) The empty set is a subset of \( D \).
      (iv) The empty set is a subset of \( E \).
      (v) \( C \) is a subset of \( E \).