Complete all of the following problems and hand in your solutions to your tutor by the due date and time. **Make sure that your name and student number are on each sheet of your answers.** Solutions to all the problems will be distributed later. Completing 5 out of the 6 assignments is compulsory and each of the five assignments will contribute 4% towards your final grade. Late assignments will not be accepted unless you have a sufficient documented reason, such as illness.

1. Write each of the following statements in words. Determine if each statement is true or false. Write the negation of each statement in symbolic form.

   (i) \( \forall x \in \mathbb{R}, \exists y \in \mathbb{R} \) such that \( x \geq y \).

   (ii) \( \exists z \in \mathbb{Z} \) such that \( \forall x \in \mathbb{R}, \forall y \in \mathbb{R}, x + y \geq z \).

2. Arthur wants to sell a new range of uni-sex lingerie: *Snake Skin Slinkies, Luscious Leather Lovelies, Purple Passion Pants* and *Raunchy Rubber Rompers*. He first needs to undertake some market testing, so arranges for five people (Boris, Doris, Noris, Morris and Horace) to watch him model each item, and he observes whether each person likes that item or not.

   Doris likes the Lovelies and the Rompers. Boris likes the Rompers and the Slinkies. Noris likes the Slinkies, Rompers and Lovelies. Horace only likes the Pants. Morris has a headache and likes none of them.

   Let \( P \) be the set of all people (that is, the five people watching Arthur), let \( L \) be the set of lingerie, and let \( e(x, y) \) mean person \( x \) likes lingerie \( y \). Consider each of the statements (a) to (f):

   (a) At least one person likes an item of lingerie.

   (b) Somebody likes all items of lingerie.

   (c) There is an item of lingerie which nobody likes.

   (d) Everybody likes some item of lingerie.

   (e) There is an item of lingerie which everybody likes.

   (f) Everybody has an item of lingerie which they do not like.

For each of the statements (a) to (f):

   (i) Write the statement using symbols.

   (ii) Write the negation of the statement in english.

   (iii) Write the negation of the statement in symbols.

   (iv) Identify which of the statement or its negation is true, and explain why.

   (continued over...)
3. Complete the following questions from the textbook:

- 2nd edition: Section 3.6, page 161: Questions 8

or

- 3rd edition: Section 3.1, pages 139-141: Questions 6, 42, 47.
- 3rd edition: Section 3.4, pages 163-164: Questions 31a
- 3rd edition: Section 3.6, page 161: Questions 18

4. Do each of the following:

(a) Use the unique factorisation theorem to write 2100 in standard factored form.

(b) If \( n = -37 \) and \( d = 9 \), find integers \( q \) and \( r \) such that \( n = dq + r \) and \( 0 \leq r < d \).

(c) Evaluate \( 28 \div 5 \) and \( 28 \mod 5 \).

5. Each of the following “proofs” is incorrect. Explain why each “proof” is wrong.

(a) The product of any four consecutive integers is divisible by 4.
   Proof: Consider the four consecutive integers 1, 2, 3, 4. The product
   \[ 1 \cdot 2 \cdot 3 \cdot 4 = 24. \]
   Now 24 is divisible by 4. Hence product of four consecutive integers is divisible by 4.

(b) For all integers \( a, b, c \), if \( a \mid bc \), then \( a \mid b \).
   Proof: Let \( a, b, c \) be integers and suppose that \( a \mid b \). Then \( b = ra \) for some integer \( r \). Multiplying both sides of this equation by \( c \) we get
   \[ bc = (ra)c = (rc)a. \]
   Since \( rc \) is an integer, we know that \( a \mid bc \). Hence if \( a \mid bc \), then \( a \mid b \).
(c) The difference of any two odd integers is even.

**Proof:** Let \( m \) and \( n \) be odd integers. Suppose that \( m - n \) is even. Thus \( m - n = 2r \) for some integer \( r \). Now since \( m \) and \( n \) are both odd, we have

\[
m = 2s + 1 \quad \text{for some integer} \quad s,
\]

and

\[
n = 2t + 1 \quad \text{for some integer} \quad t.
\]

Thus

\[
m - n = (2s + 1) - (2t + 1) = 2r,
\]

so the difference of any two odd integers is even.

6. Use the Euclidean Algorithm to calculate the greatest common divisor of each of the following pairs of numbers.

   (a) 115, 45
   
   (b) 1050, 240

7. Use your result from Question 6 to find a solution to the linear Diophantine equation \( 1050c + 240d = 600 \).

8. Use your result from Question 6 to show that there does not exist a point with integer co-ordinates that lies on the line \( 115x + 45y = 9 \).