**The University of Melbourne–Department of Mathematics and Statistics
School Mathematics Competition, 2016**

JUNIOR DIVISION

Time allowed: Two hours

These questions are designed to test your ability to analyse a problem and to express yourself clearly and accurately. The following suggestions are made for your guidance:

(1) *Considerable weight will be attached by the examiners to the method of presentation of a solution. Candidates should state as clearly as they can the reasoning by which they arrived at their results. In addition, more credit will be given for an elegant than for a clumsy solution.*

(2) *The **six** questions are not of equal length or difficulty. Generally, the later questions are more difficult than the earlier questions.*

(3) *It may be necessary to spend considerable time on a problem before any real progress is made.*

(4) *You may need to do considerable rough work but you should then write out your final solution neatly, stating your arguments carefully.*

(5) *Credit will be given for partial solutions; however a good answer to one question will normally gain you more credit than sketchy attempts at several questions.*

*Textbooks, electronic calculators and computers are **NOT** allowed. Otherwise normal examination conditions apply.*

1. Queen Mab. The wicked Queen Mab is mixing a magic potion in order to put a spell on Lancelot. She has two large cauldrons, one containing 9 litres of dragon blood and the other containing 6 litres of unicorn blood. She pours 3 litres of the dragon blood into the cauldron containing the unicorn blood. She thoroughly stirs, and then pours 3 litres of the mixture back into the cauldron with dragon blood. How much dragon blood is in each of the cauldrons, assuming Queen Mab has not spilled a drop of blood?

2. The Panama papers. On April 3 the so-called "Panama papers" were leaked, detailing the shady tax affairs of many very wealthy people, including prime ministers, arms dealers and celebrities. A man who called himself "John Doe" stole the papers, contained in three large boxes, from the firm Mossack Fonseca in Panama City. Together with his trusted helper, "Jane Roe", he brought them to safety in neighbouring Costa Rica. To avoid being discovered crossing the border, John and Jane used a 20 km hidden track through the mountains. Because the track was very steep, they could only carry one box each at a time and walk at a constant pace of 1 km per hour (whether carrying a box or not). What is the shortest time it could have taken John and Jane to carry the three boxes from the start to the finish of the track?

3. The Donald. US presidential hopeful Donald Trump's most prized possession is his toupée. There is always a lot of hot air blowing around Mr Trump, so for his upcoming tour around the United States he has decided to bring plenty of spare hairpieces, taking with him n containers, each containing 1000 toupées. Each container is carried on board his private jet by exactly 7 of his personal assistants (PAs), and each of his PAs helps carry exactly 10 containers. If Mr Trump has at least 65 and at most 70 PAs, how many toupées is he taking with him on his tour?

4. A knight's tale. To reach his beloved Guinevere, Lancelot must defeat 11 hideous monsters. Unfortunately, the evil Queen Mab has put a spell on Lancelot, and he can only make knight-moves (two down and one right/left, one down and two right/left, one up and two right/left, or two up and one right/left). Can he defeat all 11 monsters and reach Guinevere without occupying each square more than once (and without leaving the kingdom, consisting of the 13 squares)? If your answer is yes, show a way Lancelot can reach Guinevere. If your answer is no, explain clearly why Lancelot must fail.
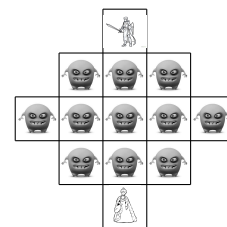
5. The Rio Olympics. At the upcoming Rio Olympic Games there will be two new sports: rugby sevens and golf. In golf, countries will compete against each other in teams of 4 players. Each player in a team plays 4 holes, and the total score of a team is the sum of the score of the individual players. For example, if the players take 8, 6, 4 and 5 shots respectively to complete their 4 holes, then the team's score is $8 + 6 + 4 + 5 = 23$. Recently, Australia played an Olympic qualifier against New Zealand. Australia's total score was 30, and New Zealand's was 32. Of the eight players, no two achieved the same score. The best (= lowest) scoring player was from New Zealand while the worst (= highest) scoring player was from Australia. What were the scores of the 4 Australian players?

6. The Melbourne Cup. Melbourne's Flemington racecourse is 2300 metres long. Horses Prince Of Penzance, Protectionist, Fiorente, Green Moon, Dunaden, Americain, Shocking, Viewed, Efficient, Delta Blues and Makybe Diva are each placed at a random but different position along the race track. At the sound of the starter's pistol, all horses start running at a constant speed of 72 kilometres per hour.

Depending on the direction in which a horse is facing, it runs towards the finish line in either a clockwise or anti-clockwise direction. There is a twist in the race, however. Whenever two horses meet along the course, they both turn around and start running towards the finish line in the opposite direction (still at the same speed of 72 kilometres per hour). How long must you wait before you know with 100% certainty that all horses have crossed the finish line? Give your answer in seconds.