THE UNIVERSITY OF
MELBOURNE

The University of Melbourne/Statistical Consulting Centre
School Mathematics Competition 2011

JUNIOR DIVISION
Time allowed: Two hours

These questions are designed to test your ability to analyse a problem and to express yourself clearly and accurately. The following suggestions are made for your guidance:

(1) *Great weight will be attached by the examiners to the method of presentation of a solution. Candidates should state as clearly as they can the reasoning by which they arrived at their results. In addition, more credit will be given for an elegant than for a clumsy solution.*

(2) *The **six** questions are not of equal length or difficulty. Generally, the later questions are more difficult than the earlier questions.*

(3) *It may be necessary to spend considerable time on a problem before any real progress is made.*

(4) *You may need to do considerable rough work but you should then write out your final solution neatly, stating your arguments carefully.*

(5) *Credit will be given for partial solutions; however a good answer to one question will normally gain you more credit than sketchy attempts at several questions.*

*Textbooks, electronic calculators and computers are **NOT** allowed. Otherwise normal examination conditions apply.*

1. The Bev. Frendan Bevola, desperate for some extra cash, borrows money from his estranged wife Alex and decides to try his luck at Crown Casino. Playing roulette he wins his first bet, doubling the money he borrowed from Alex. He isn't so lucky in his next bet, losing $10,000. He then has another win, once again doubling his money, and another loss, costing him $10,000. After doubling his money for a third time Bev loses $10,000 for a final time, leaving him without a single cent.
How much money did The Bev borrow from Alex?

2. Year of the rabbit. It is a little-known fact that former Melbourne Lord Mayor John So started his professional career as a teacher at Fitzroy High School. Thanks to his legendary maths puzzles he was voted most popular teacher at the school—a feat he later reprised in his role as Lord Mayor, when he became the world's most popular Mayor. From the school archives the competition organisers have unearthed the following puzzle, which John So prepared to celebrate Chinese New Year.

Consider a triangle consisting of ten numbers, four in the top row, three in the second row, two in the third row and one in the bottom row, such that each number in the bottom three rows is formed by summing up the two numbers directly above it. An example of such a triangle is

$$\boxed{10} \quad \boxed{3} \quad \boxed{1} \quad \boxed{8}$$
$$\boxed{13} \quad \boxed{4} \quad \boxed{9}$$
$$\boxed{17} \quad \boxed{13}$$
$$\boxed{30}$$
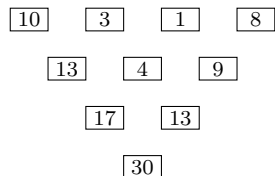
Complete the following triangle according to the same pattern:

$$\boxed{20} \quad \boxed{} \quad \boxed{} \quad \boxed{11}$$
$$\boxed{} \quad \boxed{} \quad \boxed{}$$
$$\boxed{} \quad \boxed{}$$
$$\boxed{2011}$$
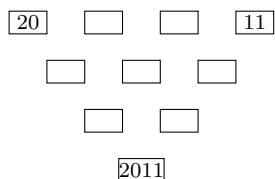
Hint: There is more than one possible solution.

3. Lady Gaga. Lady Gaga's soon-to-be-released single "Mathematical Oddity" is a compilation of five of her best-known songs. In the table below you can find the duration (in seconds) of each of the five songs that make up "Mathematical Oddity".

	Just Dance	Poker Face	LoveGame	Paparazzi	Bad Romance
Duration of song	93	84	82	78	73

The mathematical oddity of Lady Gaga's new single is that if you play the first 2 songs only, then the average length of these two songs is a whole number of seconds. If that's not odd enough, if you play the first 3 songs only, then the average length of these three songs is also a whole number of seconds. Even more bizarrely, if you play the first 4 songs only, then the average length of these four songs is also a whole number of seconds. Of course, if you play all 5 songs, the average length is also a whole number, since $(93 + 84 + 82 + 78 + 73)/5 = 82$.
Which of the 5 songs is played last on "Mathematical Oddity"?

4. Never trust your teacher. Your maths teacher is extremely excited. Using four different shapes, as shown in the two pictures below he believes to have irrefutably proven that $32.5 = 31.5$.
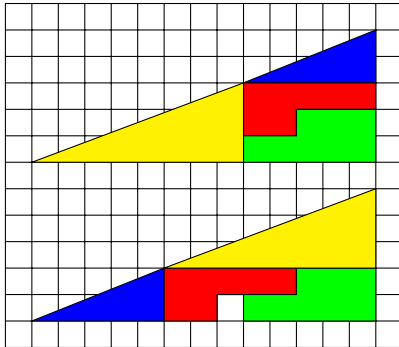
Area coloured by shapes $= \frac{13 \times 5}{2} = 32.5$

Area coloured by shapes $= \frac{13 \times 5}{2} - 1 = 31.5$

Explain clearly why, as usual, your teacher is wrong.

5. Friend or foe? A small function at Parliament House is attended by the following twelve people: Tony Abbott, Julie Bishop, Bob Brown, Julia Gillard, Joe Hockey, Barnaby Joyce, Rob Oakeshott, Tanya Plibersek, Christopher Pyne, Kevin Rudd, Wayne Swan and Malcolm Turnbull. These politicians, not being the best of friends, only shake hands with attendees whom they do not openly despise.

Tony shakes hands with one person, Julie with two, Bob with three, Julia with four, Joe with five, Barnaby with six, Rob with seven, Tanya with eight, Christopher with nine, Kevin with ten and Wayne with eleven.

With whom does Malcolm shake hands?

6. Fun run. The organisers of a fun run in the CBD have been given a number of streets by the Melbourne City Council which they can use for the run, as shown on the map below:
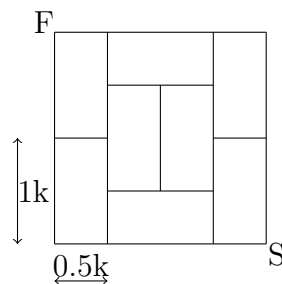
It is decided to start the race at S and finish at F, but a discussion breaks out among the organisers as to what the longest possible route from S to F is. All flunked their year 12 maths exam, and you are called in to settle the issue.

Find the length of the longest route from S to F (in kilometres) such that no section of street is used more than once, and show that no longer route exists, ending all discussion among the clueless organisers.

Each of the eight city blocks shown on the map are the same size: 0.5 by 1 kilometres.