# MATH2400: Mathematical Analysis

First Semester 2012
01/05/2012

## Assignment Number 5

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**Problem 1** *(6 points)*
Let $\Omega$ be an interval in $\mathbb{R}$. Recall that a function $f$ is called *Lipschitz continuous* on $\Omega$ with Lipschitz constant $L$ if there holds

$$|f(x) - f(y)| \leq L|x - y| \qquad \text{for all } x, y \in \Omega.$$

(a) Show that the composition of Lipschitz continuous functions is again Lipschitz continuous.
(b) Is the pointwise maximum of two Lipschitz continuous functions necessarily Lipschitz continuous?
(In each case you will need to formulate an appropriate statement, and then provide a proof (or maybe a counterexample, in (b)).

**Problem 2** *(6 points)*
(a) Suppose that $f$ is differentiable on $[2, 4]$ and is such that $f(2) = 6$ and $f' \geq 2$ on $[2, 4]$. Find the lowest possible value of $f(4)$.
(b) Give a function $f$ that is differentiable on $\mathbb{R}$ and which has the following 3 properties, or explain why such a function can't exist: $f(1) = 1$, $f(3) = 6$, and $f'(x) \leq 2$ for all $x$.

**Problem 3** *(8 points)*
(a) Prove by induction: $\displaystyle\sum_{k=1}^{n} k^2 = \frac{1}{6}n(n+1)(2n+1)$.
(b) By explicitly calculating upper and lower Riemann sums on a uniform partition and passing to the limits, calculate $\displaystyle\int_0^1 x^2\,dx$. (Hint: part a) wasn't just there for fun.)

Due: 11:50 AM, Wednesday, 16/05/2012.

Current assignments will be available at
`http://www.maths.uq.edu.au/courses/MATH2400/Tutorials.html`