The University of Melbourne–Department of Mathematics and Statistics
School Mathematics Competition, 2013

JUNIOR DIVISION
Time allowed: Two hours

These questions are designed to test your ability to analyse a problem and to express yourself clearly and accurately. The following suggestions are made for your guidance:

(1) *Considerable weight will be attached by the examiners to the method of presentation of a solution. Candidates should state as clearly as they can the reasoning by which they arrived at their results. In addition, more credit will be given for an elegant than for a clumsy solution.*

(2) *The **six** questions are not of equal length or difficulty. Generally, the later questions are more difficult than the earlier questions.*

(3) *It may be necessary to spend considerable time on a problem before any real progress is made.*

(4) *You may need to do considerable rough work but you should then write out your final solution neatly, stating your arguments carefully.*

(5) *Credit will be given for partial solutions; however a good answer to one question will normally gain you more credit than sketchy attempts at several questions.*

*Textbooks, electronic calculators and computers are **NOT** allowed. Otherwise normal examination conditions apply.*

1. Averaging. Fill the 3×3 square

with the numbers 1 to 9 such that for each horizontal, vertical and diagonal triplet of numbers, the number in the middle is the average of the numbers on either side.

2. Regular polygons. You are given an equilateral triangle of side length 2 and a regular hexagon of side length 1:
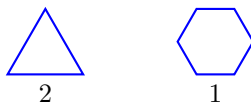
Find the ratio of their respective areas.

3. Year of the snake. Julia Gillard and Tony Abbott attend a party hosted by their mutual friend Kevin Rudd. Both have too much to drink and decide to walk home to clear their heads. They struggle to keep on course, however, and Julia snakes home

like this: whereas Tony snakes home like this: where R denotes Rudd's place.
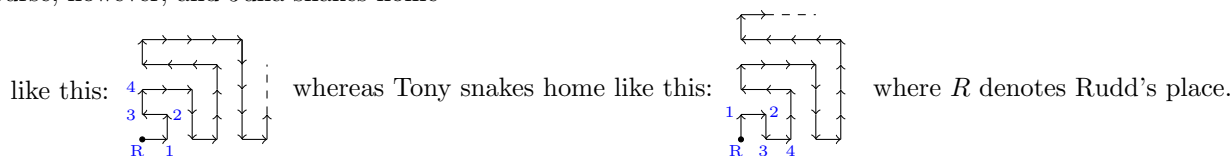
Julia and Tony each take one step of one metre per second, going from R to 1 with their first step, from 1 to 2 in their next step, and so on. Tony leaves Rudd's house two seconds after Julia but bumps into her after just one step. Both being tipsy they view each other with 'fresh' eyes; Tony is suddenly thrilled by Julia's fiery red locks and Julia really likes the look of Tony's powerful ears. As a result they share a forbidden kiss, and then continue on their respective walks. Soon, however, they meet again and another guilty kiss follows. Their passion continues till they both reach home after exactly 2013 steps.
How many kisses do they share?

4. Colour coding. You are going on a school camp supervised by the maths teaching staff. Because maths teachers are incredibly good at recognising patterns but hopeless with names, your principal has come up with the following plan. Every student has to wear shoes, a pair of socks, shorts, a t-shirt and a hat, such that each of these 5 items of clothing is in one of the three school colours. Moreover, every student has to wear at least one item of each colour, and no two students are allowed to wear exactly the same clothing-colour combination. What is the maximal number of students that can go on camp?

5. Not so popular. Our lovers, Julia and Tony, are equally popular on the first of May 2013. In 136 days time—on the 14th of September—elections will be held, and both Julia and Tony are campaigning hard to win your vote. Unfortunately, neither their policies nor their personalities are very appealing and they both lose support at a constant rate with Julia losing voters twice as fast as Tony. On election day, Julia gets a third of the number of votes that Tony gets. On what date would Julia have received two thirds of the number of votes that Tony would get?

6. Busted. Channel Nine's 2008 TV series *Underbelly* gave an account of the deadly 'gangland war', featuring some of Melbourne's most notorious criminals such as Carl "Fat Boy" Williams and Tony "The Wig" Mokbel. Sadly, it never told the true story of how Mokbel was caught importing illicit drugs from Sicily by hiding his merchandise inside one of 12 cricket balls he carried in a suitcase. The customs officer at Melbourne airport who checked Mokbel's wares was a former Maths Comp winner and very smart. She realised that if one of the balls had been tampered with it would not weigh the same as the other 11 balls. By using a set of balance-scales

no more than three times she quickly identified the ball containing drugs.
Show that someone clever can always do this using the scales no more than three times.