The University of Melbourne School Mathematics Competition, 2010

JUNIOR DIVISION
Time allowed: Two hours

These questions are designed to test your ability to analyse a problem and to express yourself clearly and accurately. The following suggestions are made for your guidance:

(1) *Considerable weight will be attached by the examiners to the method of presentation of a solution. Candidates should state as clearly as they can the reasoning by which they arrived at their results. In addition, more credit will be given for an elegant than for a clumsy solution.*

(2) *The **six** questions are not of equal length or difficulty. Generally, the later questions are more difficult than the earlier questions.*

(3) *It may be necessary to spend considerable time on a problem before any real progress is made.*

(4) *You may need to do considerable rough work but you should then write out your final solution neatly, stating your arguments carefully.*

(5) *Credit will be given for partial solutions; however a good answer to one question will normally gain you more credit than sketchy attempts at several questions.*

*Textbooks, electronic calculators and computers are **NOT** allowed. Otherwise normal examination conditions apply.*

1. Awesome foursome. Fill out the following 4×4 square such that each row and column contains the numbers $1, 2, 3$ and 4 exactly once, and such that the sum of the numbers in each region bounded by solid lines is as indicated by the number shown in the top-right corner.

2. Masterpieces from Paris. The recently closed exhibition "Masterpieces in Paris", held at the National Gallery of Australia in Canberra and showing paintings by masters such as van Gogh, Gauguin and Cézanne, charged a $30 admission fee. Being less cultured and more stingy than Victorians, the people of Canberra thought this too expensive and visitor numbers were disappointingly low. After reducing the admission fee, the number of daily visitors went up by 50% while the museum's earnings went up by 25%.

What was the new admission fee for the exhibition?

3. Grumpy. Snow White is having peas for dinner. Being a princess, she does not like peas very much, so instead of finishing her meal, she starts counting the number of peas left on her plate. She asks the dwarfs to make guesses about this number. Sleepy guesses that the number of peas is a multiple of 10 (i.e., the number of peas is one of $10, 20, 30, \ldots$). Dopey guesses that the number of peas is a multiple of 12, Happy guesses that the number of peas is a multiple of 15, Sneezy guesses that the number of peas is a multiple of 18, and, finally, Doc guesses that the number of peas is a multiple of 30. Grumpy, who does not like games, spoils it for everyone. He has quickly counted the number of peas on Snow White's plate and says "only two of you guessed correctly".

Which of the two dwarfs made a correct guess?

4. Doing a Barnaby. During his short stint as Shadow Finance Minister, Barnaby Joyce displayed a curious grasp of numbers. One of his most infamous exclamations was "All this billions, quillions, Brazilians."

Less well known was his tax-reform proposal, recommending that if you earn x per day, you are taxed $x\%$ of that amount (with the understanding that you pay 100% tax if you earn $100 or more per day. Under Senator Barnaby's plan, which amount x gives you the highest earnings after tax has been taken out?

5. Year of the Tiger. Exactly 100 people sit around a very large dinner table at the world's most expensive restaurant, El Bulli in Spain. Every diner may order any number of dishes with the one condition: the combined bill of a diner and his/her two neighbours must exactly be 2010 Euros.

Show that each diner must spend exactly 670 Euros.

6. Hospital beds. When Kevin Rudd recently visited the Royal Melbourne Children's hospital to promote his health-care reform plans, one of the nurses tried to test Kevin's knowledge of hospital bed numbers. She told him there are two types of hospital beds, the cheap ones, which desperately need replacement, with only 4 legs, and the expensive, much more comfortable ones, with a 5th leg in the middle for extra support. She also told Kevin that in ward X there are **beds of both types** with a total of n legs and asks him to figure out how many beds there are. After a quick phone call to Minister for Education Julia Gillard, Kevin triumphantly declared

> "There are [correct number deleted] beds in the ward. In fact n is the smallest number such that, had there been $n + 1$ legs in the ward, Julia, errh, I mean I, would not have been able to tell you the number of beds".

Find the number of beds in ward X.