The University of Melbourne School Mathematics Competition, 2009

JUNIOR DIVISION
Time allowed: Two hours

These questions are designed to test your ability to analyse a problem and to express yourself clearly and accurately. The following suggestions are made for your guidance:

(1) *Considerable weight will be attached by the examiners to the method of presentation of a solution. Candidates should state as clearly as they can the reasoning by which they arrived at their results. In addition, more credit will be given for an elegant than for a clumsy solution.*

(2) *The **six** questions are not of equal length or difficulty. Generally, the later questions are more difficult than the earlier questions.*

(3) *It may be necessary to spend considerable time on a problem before any real progress is made.*

(4) *You may need to do considerable rough work but you should then write out your final solution neatly, stating your arguments carefully.*

(5) *Credit will be given for partial solutions; however a good answer to one question will normally gain you more credit than sketchy attempts at several questions.*

*Textbooks, electronic calculators and computers are **NOT** allowed. Otherwise normal examination conditions apply.*

1. Walkathon. Barnaby, Julia, Kevin & Malcolm enter a parliamentary walkathon to raise money for the victims of the Victorian bush fires. Barnaby runs 1 km in 4 minutes, Julia walks that distance in 8 minutes, Kevin in 12 and Malcolm in 16 minutes. Assuming they all start together, how long will it take before the *combined distance* travelled by the fearless four totals 25 km?

2. Of course ... Due to the global financial downturn, school budgets across the globe have been severely cut. Your school in particular is in deep financial strife and the principal has decided to make students pay for their own lockers. Because the high-numbered lockers at your school are the most popular (the least number of stairs to climb) each student is charged $10 per digit on their locker. The lockers are numbered consecutively, starting with 1. The total revenue raised from charging for lockers is $69,290. How many students are at your school?

2. Size matters. Melbourne's troubled *Southern Star* giant Ferris wheel has six cabins, equally spaced around the outside of the wheel. The distance (measured along a straight line) between adjacent cabins is exactly 60 metres. Determine whether the *Southern Star* is bigger than the *London Eye* which has a diameter of 135 metres.

3. Card magic. The numbers 1 to 10 are written on 10 different cards, each card containing exactly one number. Three cards are put face-down on a table such that they are in increasing order, from left to right. The sum of the three unknown numbers on the table is 13.
Tony picks up the left-most card and says "I cannot determine what the other two cards are". Then Paul picks up the right-most card and says "I cannot determine what the other two cards are". Finally Ole picks up the middle card and says "I cannot determine what the other two cards are".
Tony, Paul and Ole, who have heard each other's claims but have not actually seen the cards picked up by the others, are all maths professors, so you can safely believe that their statements are correct. Find the value of the card in the middle.

5. Year of the Ox. The number 2009 can be written as the sum of two consecutive whole numbers: $2009 = 1004 + 1005$. Find all other ways to write 2009 as the sum of consecutive, positive whole numbers.

6. Footy. You play in an inter-school footy competition. Curiously, in one of the rounds the total number of points scored by each team is the same, so that all games are not only drawn, but also have the same final score. In that same round your team scored 1/13th of all goals and 1/15th of all behinds. How many teams play in the competition?
(For those from NSW, a goal is worth 6 points and a behind 1 point.)