INTERNAL STUDENTS ONLY

THE UNIVERSITY OF QUEENSLAND

CANDIDATES MUST NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

Second Semester Examination, November, 1999

MP213

ORDINARY DIFFERENTIAL EQUATIONS II

(Unit Courses)

Time: **TWO hours** for working

Ten minutes for perusal before examination begins

CREDIT WILL BE GIVEN ONLY FOR WORK WRITTEN ON THIS EXAMINATION SCRIPT.
Use the back pages if the space provided is insufficient, or for rough working.
Questions carry the number of marks shown. The total number of marks is 80.
All questions may be attempted.
Check that this examination paper contains pages 1 to 14 including the cover sheet (page 1) and a list of Laplace transforms (page 14).
Pocket calculators not allowed.

FAMILY NAME (PRINT): ________________

GIVEN NAMES (PRINT): ________________

STUDENT NUMBER: | | | | | | | – | | | |

SIGNATURE: ________________

EXAMINER'S USE ONLY	
QUESTION	MARK
1	
2	
3	
4	
5	
TOTAL	

1. (a) Find the solution of the initial value problem

$$y' + 3y = 3t^2 e^{-3t} \quad , \quad y(0) = 1.$$

(6 marks)

(b) Solve the initial value problem

$$y'' + 2y' + y = 4e^t + 2\cos t, \quad y(0) = 2\,, \quad y'(0) = 2.$$

(14 marks)

2. Solve the initial value problem

$$\mathbf{x}' = \begin{bmatrix} -2 & 1 \\ -5 & 4 \end{bmatrix} \mathbf{x} \quad + \quad e^t \begin{bmatrix} 4 \\ 2 \end{bmatrix} \quad , \quad \mathbf{x}(0) = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

(16 marks)

3. Find all the critical points of the nonlinear system below and determine their stability.

$$\mathbf{x}' = \begin{bmatrix} 0 & 1 \\ 4 & -1 \end{bmatrix} \mathbf{x} \quad + \quad \begin{bmatrix} 0 \\ -x_1^3 \end{bmatrix} .$$

(12 marks)

4. State the first shift theorem for Laplace transforms. What is the Laplace transform of $e^{at}\cos t$? Use Laplace transforms to solve the initial value problem

$$y'' - 2y' - 3y = 17e^{-t}\cos t, \quad y(0) = 2, \quad y'(0) = 2.$$

(15 marks)

5. (a) Find the Laplace transform of $te^{3t}\sin t$.

(6 marks)

(b) Define the unit step function $u(t-a)$. State the second shift theorem for the Laplace transform. Use Laplace transforms to solve the initial value problem

$$y' + y = 2u(t-1) \quad y(0) = 0.$$

Give a rough sketch of the solution for $t > 0$.

(11 marks)