INTERNAL STUDENTS ONLY

THE UNIVERSITY OF QUEENSLAND

CANDIDATES MUST NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

Sample Semester Examination, September, 1999

MP307

Optimization Theory III

Time: TWO hours for working

Ten minutes for perusal before examination begins

FULL WORKING MUST BE SHOWN.
Answer **ALL** the questions.
All questions carry the same number of marks.
Pocket calculators allowed.

Question 1

(a) Find the local minima and maxima of

$$x_1x_2 + 8/x_1 + 1/x_2 \ .$$

Is there a global minimum or maximum?

(b) Find and test for nondegeneracy the critical points of

$$f(x_1, x_2, x_3) = x_1x_2 + x_2x_3 + x_3x_1,$$

subject to the constraint $x_1 + x_2 + x_3 = 1$. Determine the local minimum of f.

Question 2

(i) Find the extremal of

$$J[x] = \int_0^{\pi/2} (x^2 - \dot{x}^2 - 2x\sin t)\,dt,$$

with end conditions $x(0) = 1,\ x(\pi/2) = 2$.

(ii) Find the extremal of

$$J[x] = \int_0^T \frac{\dot{x}^2}{t^3}\,dt,$$

when $x(0) = 1$ and $x(T)$ lies on the curve $x(t) = 2 + (t-1)^2$.

Question 3 on next page.

Question 3

(i) Explain what is meant by a corner condition.

(ii) Find two distinct nonsmooth extremals for

$$J[x] = \int_0^2 \dot{x}^2(\dot{x}-1)^2\,dt, \ \ x(0)=0, \ \ x(2)=1.$$

Question 4

(a) Use the Lagrange multiplier technique to find the extremals of

$$J[x] = \int_0^\pi \dot{x}^2\,dt,$$

with $x(0)=0, \ x(\pi)=0$ subject to the constraint $\int_0^\pi x^2\,dt = \pi/2$. Show that there are an infinite set of extremals. Evaluate the functional on a typical extremal.

(b) Find the extremal of

$$J[x] = \int_0^2 \left(\frac{1}{2}\dot{x}^2 + x\dot{x} + x + \dot{x}\right)\,dt,$$

with endpoint conditions $x(0)=0, \ x(2)=2$. Use the Weierstrass excess function to show that the extremal is a minimizing curve.

Question 5

Solve the problem of time-optimal control to the origin for

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -2x_1 - 3x_2 + u, \ \ |u| \le 1. \end{aligned}$$

Question 6

The system $\dot{x}_1 = -x_1 + u$, where $u = u(t)$ is not subject to a constraint, is to be controlled from $x_1(0) = 1$ to $x_1(t_1) = 2$, where t_1 is unspecified, in such a way that

$$J = \frac{1}{2}\int_0^{t_1} (x_1^2 + u^2)\,dt$$

is minimized. Find the optimal control.