Assignment Asterisked Questions

MATH2010

Tutorial Sheet 2 - Week 3

1. Find the general solutions of the following nonhomogeneous systems

   $*$(i)

$$x' = \begin{pmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix} x + \begin{pmatrix} e^t \\ \sqrt{3}e^{-t} \end{pmatrix}.$$

   (ii)

$$x' = \begin{pmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \end{pmatrix} x + \begin{pmatrix} e^t \\ 1 \\ 1 \end{pmatrix}.$$

2. Find the transfer function of the control system

$$x' = \begin{pmatrix} -2 & 2 \\ 1 & -1 \end{pmatrix} x + (1\ 0)u$$

$$y = (1\ 0)x$$

3. Find the matrix transfer function corresponding to

$$A = \operatorname{diag}(1, -1, -2),\ B = \begin{pmatrix} 7/6 & 1 \\ -7/2 & -2 \\ 10/3 & 2 \end{pmatrix},\ C = \begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 2 \end{pmatrix}.$$

4. If $\{A_1, B_1, B_1\}$ and $\{A_2, B_2, B_2\}$ are realizations of matrix transfer functions $G_1(s)$ and $G_2(s)$ respectively, find a realization of $G_1(s)G_2(s)$, assuming that all matrix products exist.

5. Sketch the trajectories corresponding to the solutions of the following initial value problems:

   (a)

$$\frac{dx}{dt} = -x, \quad \frac{dy}{dt} = 2y, \quad x(0) = 4,\ y(0) = 2.$$

   $*$(b)

$$\frac{dx}{dt} = -y, \quad \frac{dy}{dt} = x, \quad x(0) = 4,\ y(0) = 0.$$

6. For systems (i) and (ii) below, (a) find the eigenvalues and eigenvectors,

   (b) classify the critical point $(0,0)$ as to type and determine whether it is stable, asymptotically stable or unstable.

   $*$(i)

$$x' = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix} x.$$

   (ii)

$$x' = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix} x.$$

7. In each of the systems (i) and (ii) verify that $(0,0)$ is a critical p oint, find the linear approximation and discuss the type and stability of the critical point $(0,0)$ by examining the corresponding linear system

   $*$(i)

$$\begin{aligned} x' &= (1+x)\sin y \\ y' &= -x + 1 - \cos y \end{aligned}$$

   Are there any other critical points of this system? Discuss (bonus for this extra bit!)

   (ii)

$$\begin{aligned} x' &= x + y^2 \\ y' &= x + y \end{aligned}$$