Assignment Asterisked Questions

MATH2010

Tutorial Sheet 5 - Week 6

∗1. Solve the following initial value problem

$$x' = \begin{pmatrix} 1 & 4 \\ 1 & 1 \end{pmatrix} x + \begin{pmatrix} 6t - t^2 + 3e^{2t} \\ -1 + t - t^2 + e^t \end{pmatrix}, \quad x(0) = \begin{pmatrix} 3 \\ 0 \end{pmatrix}.$$

∗2. Determine the location of all critical points, and then determine their type (saddle? focus? etc.) by linearization of the system $x' = \begin{pmatrix} x_2 \\ -4x_1 + 5x_1^3 - x_1^5 \end{pmatrix}$.

3. Using the Laplace transform, solve the following:

   ∗(a) $y'' + 4y' + 13y = 145\cos 2t, \quad y(0) = 9,\ y'(0) = 19.$ (First Shift)

   (b) $y'' + 4y = r(t),\ r(t) = 3\sin t$ if $0 < t < \pi$ and $-3\sin t$ if $t > \pi,\ y(0) = 0,\ y'(0) = 3.$ (Second Shift)

∗4. Find the inverse transforms or Laplace transforms of the following functions:

   (a) $(s+3)/((s+3)^2+1)^2$ (1st shift & differentiate transform)

   (b) $e^t u(t - 1/2)$ (2nd shift)

   (c) $\ln \frac{s^2+1}{(s-1)^2}$ (integration of what transform?)

∗5 Find the transfer function of the control system

$$x' = \begin{pmatrix} -2 & 2 \\ 1 & -1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u$$

$$y = (1\ 0)x$$

What output corresponds to an input of $u(t) = 27t$?

6 Find the matrix transfer function corresponding to

$$A = \operatorname{diag}(1, -1, -2),\ B = \begin{pmatrix} 7/6 & 1 \\ -7/2 & -2 \\ 10/3 & 2 \end{pmatrix},\ C = \begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 2 \end{pmatrix}.$$