# Assignment Asterisked Questions

## MATH2010

## Tutorial Sheet 4 - Week 5

1. (a) Sketch the following functions and find their Laplace transforms.
   **(i) $tu(t-1)$
   (ii) $u(t-\pi)\cos t$
   **(b) Using the Second Shift Theorem, solve
   $$y'' - 5y' + 6y = r(t), \ y(0) = 1, \ y'(0) = -2,$$
   where $r(t) = 4e^t$ if $0 < t < 2$ and $r(t) = 0$ if $t > 2$.

2. Use the Laplace transform to solve the initial value problem
   $$y'''' - 4y''' + 6y'' - 4y' + y = 0, \ y(0) = 0, \ y'(0) = 1, \ y''(0) = 0, \ y'''(0) = 1.$$

3. Using the differentiation formula $-F'(s) = \mathcal{L}[tf(t)]$ or the integration formula $\mathcal{L}^{-1}\left[\int_s^\infty F(\sigma)\,d\sigma\right] = f(t)/t$ to find $f(t)$ if $F(s)$ is:

   (i) $\frac{4}{(s+1)^2}$,
   **(ii) $\frac{4s}{(s^2+4)^2}$,
   (iii) $\frac{s}{(s^2+1)^2}$,
   (iv) $\ln\frac{s}{s-1}$,
   **(v) $\operatorname{arccot}(s/\omega)$,    **BONUS!!**.

4. Use the convolution of Laplace transforms, solve
   $$y(t) = t + e^t - \int_0^t y(\tau)\cosh(t - \tau)d\tau.$$

5. Sketch the following functions, which are assumed to have period $2\pi$, and find the Laplace transforms.

   **(a) $f(t) = \begin{cases} 1, & \text{if} \quad 0 < t < \pi \\ -1 & \text{if} \quad \pi < t < 2\pi. \end{cases}$

   (b) $f(t) = \begin{cases} t, & \text{if} \quad 0 < t < \pi \\ \pi - t, & \text{if} \quad \pi < t < 2\pi. \end{cases}$