INTERNAL STUDENTS ONLY

The University Of Queensland

CANDIDATES MUST NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

Special/Supplementary Examination, August, 2003

MATH2010

ANALYSIS OF ORDINARY DIFFERENTIAL EQUATIONS

(Unit Courses)

Time: **ONE HOUR** for working

TEN minutes for perusal before examination begins

CREDIT WILL ONLY BE GIVEN FOR WORK WRITTEN ON THIS EXAMINATION SCRIPT.

Answer ALL questions.

All questions carry the same number of marks.

Pocket calculators may not be used.

Check that this examination paper has 11 printed pages!

The Laplace transform table may be removed for convenience in working.

FAMILY NAME (PRINT): ____________________

GIVEN NAMES (PRINT): ____________________

STUDENT NUMBER: | | | | | | | | |

SIGNATURE: ____________________

EXAMINER'S USE ONLY	
QUESTION	MARK
1	
2	
3	
TOTAL	

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1. (a) Solve the initial value problem

$$\boldsymbol{x}' = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 2e^{2t} \\ 3e^{2t} \end{bmatrix}, \quad \boldsymbol{x}(0) = \begin{bmatrix} -2/3 \\ 1/3 \end{bmatrix}$$

Question 1 continued on next page

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(b) Consider the input-output system

$$\begin{aligned} \boldsymbol{x}'(t) &= \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{b}u(t) \\ y(t) &= \boldsymbol{c}\boldsymbol{x}(t) \\ \boldsymbol{A} &= \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \boldsymbol{c} = [1 \ \ 1], \end{aligned}$$

where $\boldsymbol{x}(t)$ is the state of the system, $u(t)$ is scalar input and $y(t)$ is scalar output. Find the transfer function of the system. When $u(t) = e^{-t}$, find the output as a function of t.

Question 1(b) working space on next page

2. (a) Find the Laplace transforms of the functions
   (i) $f(t) = 4t(u(t) - u(t-2))$,
   (ii) $f(t) = t \sin t$.

Question 2 continued on next page

2. (b) Find the the function whose Laplace transform is

$$\frac{3(s+1)}{s^2+6s+10},$$

Question 2 continued on next page

2. (c) Use the Laplace transform technique to solve the initial value problem

$$x'' - 5x' + 6x = r(t), \quad x(0) = 1,\ x'(0) = -2,$$

where $r(t) = 4e^t$ if $0 < t < 2$ and $r(t) = 0$ if $t > 2$.

Question 2 working space on next page

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3. Locate the equilibrium points of the system

$$\begin{aligned} x_1' &= x_1 - x_1^2 - x_1 x_2 \\ x_2' &= \frac{1}{2}x_2 - \frac{1}{4}x_2^2 - \frac{3}{4}x_1 x_2. \end{aligned}$$

Determine their type by linearization (*do not sketch the trajectories*).

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Table of Laplace transforms on final page