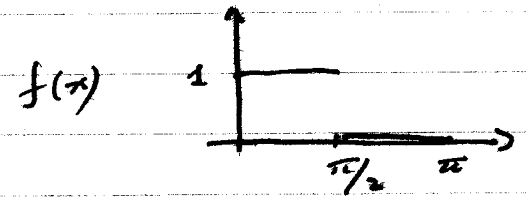
2. If the bar is insulated at both ends,

$u_x(0,t) = u_x(L,t) = 0$, $u(x,0) = f(x)$. This gives

$$u(x,t) = A_0 + \sum_{n=1}^{\infty} A_n \cos\frac{n\pi x}{L}\, e^{-(cn\pi/L)^2 t},$$

where $A_0 = \frac{1}{L}\int_0^L f(x)\,dx$

$$A_n = \frac{2}{L}\int_0^L f(x)\cos n\pi x/L \; dx$$



Here $L = \pi$, $c = 1$

$$A_0 = \frac{1}{\pi}\int_0^{\pi/2} 1 \, . \, dx = \frac{1}{2}$$

$$A_n = \frac{2}{\pi}\int_0^{\pi/2} 1.\ \cos nx \, dx = \frac{2}{n\pi}\sin\frac{n\pi}{2}$$

$$= \begin{cases} 0, & n = 2k \text{ is even} \\ \dfrac{2}{(2k-1)\pi}(-1)^{k+1}, & n = 2k-1 \text{ is odd.} \end{cases}$$

$$u(x,t) = \frac{1}{2} + \frac{2}{\pi}\left(\frac{(\cos x)e^{-tc^2}}{1} - \frac{(\cos 3x)e^{-9tc^2}}{3} + \frac{(\cos 5x)e^{-25tc^2}}{5} - \cdots\right)$$

$$= \frac{1}{2} + \frac{2}{\pi}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}\cos(2k-1)x}{(2k-1)}\, e^{-(2k-1)^2 tc^2}.$$