MT253/256 Assignment 9/4 Solutions

1. Has the same constants as the example done in lectures, $c^2 = 1.752 = K/\sigma\rho$. Now $L = 10$, so

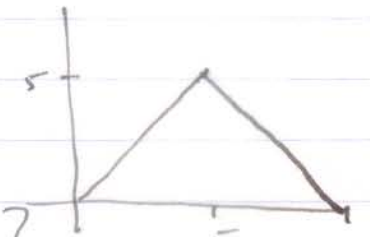
$$u(x,t) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{10}\, e^{-\lambda_n^2 t},$$

$\lambda_n = c n\pi/10$, $n = 1, 2, \ldots$, i.e. $\lambda_n^2 = 0.01752 n^2\pi^2$

$$f(x) = u(x,0) \Rightarrow \sum_{n} B_n \sin\frac{n\pi x}{10} = 5 - |x-5|.$$

So, $B_n = \frac{2}{10}\int_0^{10} f(x)\sin\frac{n\pi x}{10}\,dx$

But $5 - |x-5| = \begin{cases} x, & 0 < x < 5 \\ 10 - x, & 5 < x < 10 \end{cases}$



$$B_n = 0.2\left\{\int_0^5 x\sin\frac{n\pi x}{10}\,dx + \int_5^{10}(10-x)\sin\frac{n\pi x}{10}\,dx\right\}$$

$$= 0.2\left\{\frac{-50}{n\pi}\cos\frac{n\pi}{2} + \frac{100}{n^2\pi^2}\sin\frac{n\pi}{2} + \frac{50}{n\pi}\cos\frac{n\pi}{2} + \frac{100}{n^2\pi^2}\sin n\pi/2\right\}$$

$$= \frac{40}{n^2\pi^2}\sin\frac{n\pi}{2}$$

$$= \begin{cases} 0, & n = 2k \text{ is even} \\ \frac{40}{(2k-1)^2\pi^2}(-1)^{k+1}, & \text{if } n = 2k-1 \text{ is odd.} \end{cases}$$

$$u(x,t) = \frac{40}{\pi^2}\left[\frac{\sin 0.1\pi x\, e^{-0.01752\pi^2 t}}{1^2} - \frac{\sin 0.3\pi x\, e^{-0.01752(3\pi)^2 t}}{3^2} + \frac{\sin 0.5\pi x\, e^{-0.01752(5\pi)^2 t}}{5^2} - \cdots\right]$$

$$= \frac{40}{\pi^2}\sum_{k=1}^{\infty}\sin\frac{(2k-1)\pi x}{10}\, e^{-0.01752(2k-1)^2\pi^2 t}/(2k-1)^2$$