4. $u_{tt} = u_{xx}$, $L=\pi$, $u(0,t) = u(\pi,0) = 0 \quad \forall t$

$u(x,0) = f(x) = 0.1x(\pi - x)$, $u_t(x,0) = 0$.

Going through the same steps as in Q2,

$F_n(x) = \sin nx$, $G_n(t) = B_n \cos nt$, $B_n^* = 0$ from $u_t(x,0)=0$.

$u_n(t) = B_n \cos nt \sin nx$, $n = 1, 2, \dots$

where the B_n are the cfts of Fourier sine series of f.

$$B_n = \frac{2}{\pi}\int_0^{\pi} f(x) \sin nx \, dx.$$

$$= \frac{0.2}{\pi}\int_0^{\pi} x(\pi - x) \sin nx \, dx$$

$$= \frac{0.2}{\pi}\left[x(\pi-x)\left(-\frac{\cos nx}{n}\right)\right]_0^{\pi} + \frac{0.2}{\pi n}\int_0^{\pi}(\pi - 2x)\cos nx \, dx$$

$$= \frac{0.2}{\pi n}\int_0^{\pi}(\pi - 2x)\cos nx \, dx = \frac{0.2}{\pi n}\left[(\pi - 2x)\frac{\sin nx}{n}\right]_0^{\pi}$$

$$- \frac{0.2}{\pi n^2}\int_0^{\pi}(-2)\sin nx \, dx$$

$$= -\frac{0.4}{\pi n^2}\left[\frac{\cos nx}{n}\right]_0^{\pi} = \frac{0.4}{\pi n^3}(1 - \cos n\pi)$$

$$= \frac{0.4}{\pi n^3}\left(1 - (-1)^n\right).$$

$$B_n = \begin{cases} 0, & n \text{ even}, \ n = 2k \\ \dfrac{0.8}{\pi n^3}, & n \text{ odd}, \ n = 2k-1 \end{cases}$$

$$u(x,t) = \sum_{n=1}^{\infty} B_n \cos nt \sin nx$$

$$= \frac{0.8}{\pi}\sum_{k=1}^{\infty} \frac{\cos(2k-1)t \, \sin(2k-1)x}{(2k-1)^3}$$

$$\left(\text{OR} \quad \frac{0.8}{\pi}\left(\frac{\cos t \sin x}{1^3} + \frac{\cos 3t \sin 3x}{3^3} + \frac{\cos 5t \sin 5x}{5^3} + \dots\right)\right)$$