6 (i) $\begin{vmatrix} 3-\lambda & -2 \\ 2 & -2-\lambda \end{vmatrix} = \lambda^2 - \lambda - 2 = 0$

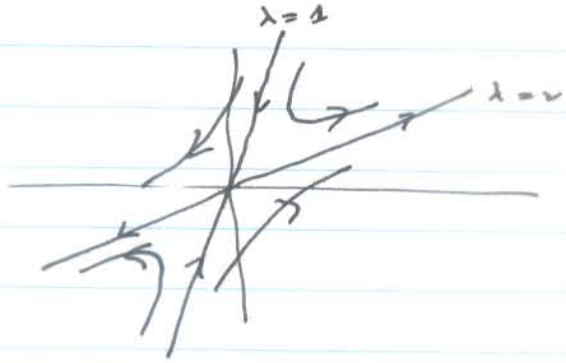
$(\lambda-2)(\lambda+1) = 0$, $\lambda = 2, -1$

$(3-\lambda)a_1 - 2a_2 = 0$

$\lambda = 2$, $a_1 - 2a_2 = 0$ $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$; $\lambda = -1$, $4a_1 - 2a_2 = 0$ $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

This is SADDLE & is unstable

A rough drawing (which you were not asked to do but I'll show you) is



7 (i) $\dot{x} = 0 = (1+x)\sin y \Rightarrow x = -1$ or $\sin y = 0$

$y' = 0 \Rightarrow -x + 1 = \cos y$. If $x = -1$, $\cos y = 2$, reject as soln.

If $\sin y = 0$, $y = n\pi$, $n = 0, \pm 1, \pm 2, \dots$ & $1 - x = \pm 1$.

If $y = 2k\pi$, $k = 0, \pm 1, \dots$, $\cos y = 1 \Rightarrow x = 0$. So $(0, 2k\pi)$ is one set of critical points (in particular, $(0,0)$)

If $y = (2k+1)\pi$, $k = 0, \pm 1, \dots$, $\cos y = -1 \Rightarrow x = 2$ & $(2, (2k+1)\pi)$ is the other set of critical points.

Linear approx at $(0,0)$ is $x' = Jx$ $\quad J = \begin{bmatrix} \partial f_1/\partial x & \partial f_1/\partial y \\ \partial f_2/\partial x & \partial f_2/\partial y \end{bmatrix}$

So $J = \begin{bmatrix} \sin y & (1+x)\cos y \\ -1 & \sin y \end{bmatrix}$. At $(0,0)$, $J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

So the lin. approx. is a centre & stable. Clearly so are all the other $(0, 2k\pi)$ type critical points.

What about $(2, (2k+1)\pi)$? All the same as $(2, \pi)$

$J = \begin{bmatrix} \sin y & (1+x)\cos y \\ -1 & \sin y \end{bmatrix}_{\substack{x=2 \\ y=\pi}} = \begin{bmatrix} 0 & -3 \\ -1 & 0 \end{bmatrix}$

Eigenvalues of J are $\pm\sqrt{3}$, SADDLE & UNSTABLE