MATH 2010/2100 SOLUTIONS 2

1(i) $\begin{vmatrix} 1-\lambda & \sqrt{3} \\ \sqrt{3} & -1-\lambda \end{vmatrix} = \lambda^2 - 4 \quad , \quad \lambda = \pm\sqrt{2}$

$(1-\lambda)a_1 + \sqrt{3}\,a_2 = 0 \qquad \lambda = 2 \Rightarrow \sqrt{3}\,a_2 = a_1 \qquad \begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix}$

$\lambda = -2 \Rightarrow 3a_1 + \sqrt{3}a_2 = 0 \quad , \quad a_1 = 1, \; a_2 = -3/\sqrt{3} = -\sqrt{3} \qquad \begin{bmatrix} 1 \\ -\sqrt{3} \end{bmatrix}$

So $\underset{\sim}{x}_h(t) = c_1 e^{2t}\begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix} + c_2 e^{-2t}\begin{bmatrix} 1 \\ -\sqrt{3} \end{bmatrix}$

Particular solution $\underset{\sim}{x}_p(t) = \underset{\sim}{u}\,e^t + \underset{\sim}{v}\,e^{-t}$

$\underset{\sim}{x}'_p = u^t - v e^{-t} = \begin{bmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{bmatrix}(\underset{\sim}{u}\,e^t + \underset{\sim}{v}\,e^{-t}) + \begin{bmatrix} e^t \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \sqrt{3}e^{-t} \end{bmatrix}$

cfts e^t $\quad \underset{\sim}{u} = \begin{bmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{bmatrix}\underset{\sim}{u} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad u_1 = -2/3, \; u_2 = -\sqrt{3}/3$

cfts e^{-t} $\quad -\underset{\sim}{v} = \begin{bmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{bmatrix}\underset{\sim}{v} + \begin{bmatrix} 0 \\ \sqrt{3} \end{bmatrix} \qquad v_1 = -1, \; v_2 = 2\sqrt{3}/3$

$$\underset{\sim}{x}(t) = \underset{\sim}{x}_h(t) + \underset{\sim}{x}_p(t)$$

$$= c_1 e^{2t}\begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix} + c_2 e^{-2t}\begin{bmatrix} 1 \\ -\sqrt{3} \end{bmatrix} + \begin{bmatrix} -2/3 \\ -\sqrt{3}/3 \end{bmatrix}e^t + \begin{bmatrix} -1 \\ 2\sqrt{3}/3 \end{bmatrix}e^{-t}$$

5(b) $\underset{\sim}{x}' = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\underset{\sim}{x} \quad , \quad \begin{vmatrix} -\lambda & -1 \\ 1 & -\lambda \end{vmatrix} = \lambda^2 + 1 = 0$

$\lambda = \pm i$

eigenvec eqn $\; -\lambda a_1 - a_2 = 0$

$\lambda = i$, $a_2 = -i a_1$ & an eigenvec is $\begin{bmatrix} 1 \\ -i \end{bmatrix}$, soln $\begin{bmatrix} 1 \\ -i \end{bmatrix}e^{it}$.
That is $\begin{bmatrix} 1 \\ -i \end{bmatrix}(\cos t + i\sin t)$. The real & imaginary parts of this solution will give two REAL, LIN. INDEPT. solutions & there is no need to consider the other, complex conjugate solution.

REAL PART $\underset{\sim}{x}^{(1)}(t) = \begin{bmatrix} \cos t \\ \sin t \end{bmatrix}$; IMAG. PART $\underset{\sim}{x}^{(2)}(t) = \begin{bmatrix} \sin t \\ -\cos t \end{bmatrix}$

$\underset{\sim}{x}(t) = c_1\underset{\sim}{x}^{(1)}(t) + c_2\underset{\sim}{x}^{(2)}(t) = \begin{bmatrix} c_1\cos t + c_2\sin t \\ c_1\sin t - c_2\cos t \end{bmatrix}$

$t = 0 \quad , \quad 4 = c_1 \quad c_2 = 0$

$\underset{\sim}{x}(t) = \begin{bmatrix} 4\cos t \\ 4\sin t \end{bmatrix}, \quad x^2 + y^2 = 16.$ Circle, centr (0, 0) radius 4.